CBSE
Class X Mathematics
Board Paper – 2017
All India Set – 3

Time: 3 hours **Total Marks: 90**

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 31 questions divided into four sections – A, B, C and D.

(iii) Section A contains 4 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.

(iv) Use of calculators is not permitted.

SECTION A

Question numbers 1 to 4 carry 1 mark each.

1. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap?

2. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun?

3. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

4. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?

SECTION B

Question numbers 5 to 10 carry 2 marks each.

5. A circle touches all the four sides of a quadrilateral ABCD. Prove that AB + CD = BC + DA

6. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

7. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, -5) is the mid-point of PQ, then find the coordinates of P and Q.

8. If the distances of P(x, y) from A(5, 1) and B(-1, 5) are equal, then prove that 3x = 2y.

9. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

10. For what value of n, are the n^{th} terms of two A.Ps 63, 65, 67,.... and 3, 10, 17,..... equal?

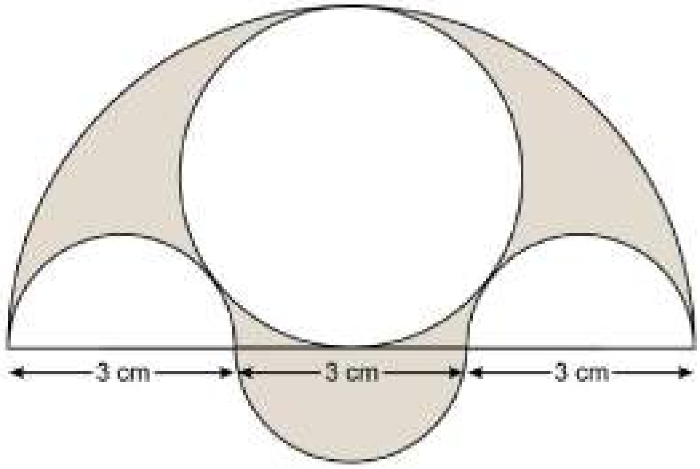
SECTION C

Question numbers 11 to 20 carry 3 marks each.

11. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

12. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.

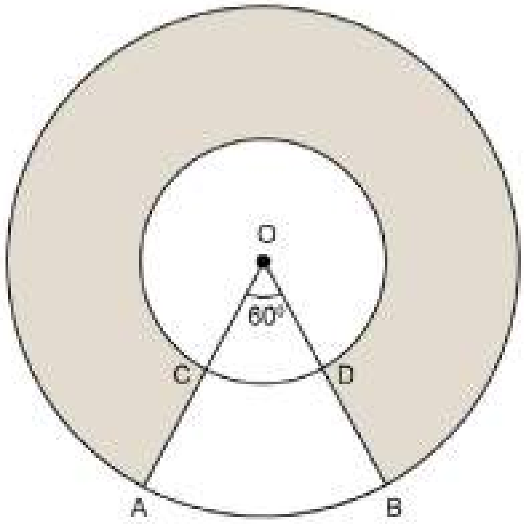
13. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



14. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divides the line segment joining the points P(2, -2) and Q(3, 7)? Also find the value of y.

15. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation?

16. In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



17. The dimensions of a solid iron cuboid are 4.4 m × 2.6 m × 1.0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

18. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

19. How many terms of an A.P. 9, 17, 25, ... must be taken to give a sum of 636?

20. If the roots of the equation $(a^2 + b^2)\, x^2 - 2(ac + bd)\, x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

SECTION D

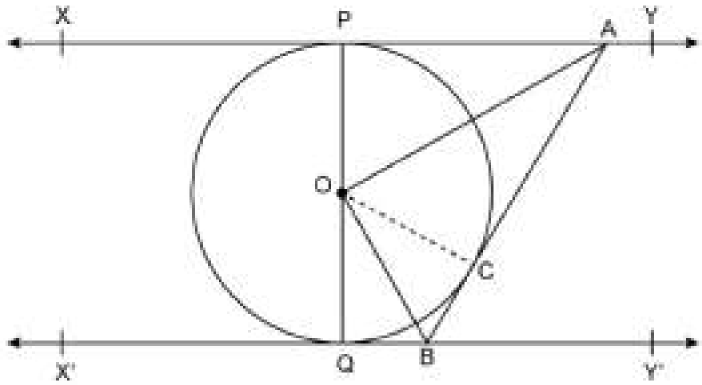
Question numbers 21 to 31 carry 4 marks each.

21. If the points $A(k + 1, 2k)$, $B(3k, 2k + 3)$ and $C(5k - 1, 5k)$ are collinear, then find the value of k.

22. Construct a triangle ABC with side BC = 7 cm, ∠B = 45°, ∠A = 105°. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the Δ ABC.

23. Two different dice are thrown together. Find the probability that the numbers obtained have
(i) even sum, and
(ii) even product

24. In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangents AB with point of contact C, is intersecting XY at A and X'Y' at B. Prove that ∠AOB = 90°.



25. In a rain–water harvesting system, the rain-water from a roof of 22 m × 20 m drains into a cylindrical tank having diameter of base 2 m and height 3.5m. If the tank is full, find the rainfall in cm. Write your views on water conservation.

26. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

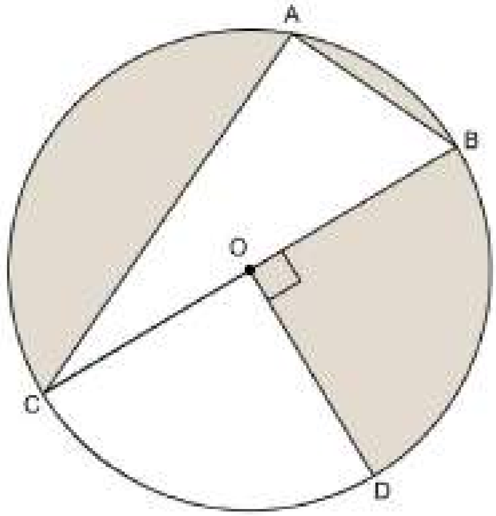
27. If the ratio of the sum of the first n terms of two A.Ps is (7n + 1) : (4n + 27), then find the ratio of their 9th terms.

28. Solve for x:
$$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2, \text{where } x \neq -\frac{1}{2}, 1$$

29. A takes 6 days less than B to do a work. If both A and B working together can do it in 4 days, how many days will B take to finish it?

30. From the top of a tower, 100 m high, a man observe two cars on the opposite sides of the tower and in same straight line with its base, with its base, with angles of depression 30° and 45°. Find the distance between the cars.
$\left[\text{Take } \sqrt{3} = 1.732\right]$

31. In the given figure, O is centre of the circle with AC = 24 cm, AB = 7 cm and $\angle BOD = 90°$. Find the area of the shaded region.



CBSE
Class X Mathematics
Board Paper – 2017 Solution
All India Set – 3

Time: 3 hours **Total Marks: 90**

SECTION A

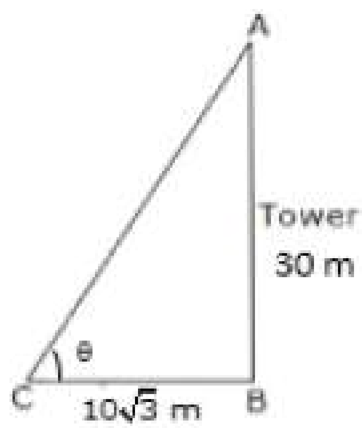
1. Let the total number of rotten apples in a heap = n
Total number of apples in a heap = 900
Probability of selecting a rotten apple from a heap = 0.18
Now,

$$\text{P(selecting a rotten apple)} = \frac{\text{Number of rotten apples}}{\text{Total number of apples}}$$

$$\Rightarrow 0.18 = \frac{n}{900}$$
$$\Rightarrow n = 0.18 \times 900$$
$$\Rightarrow n = 162$$

Hence, the number of rotten apples is 162.

2. Let AB be the tower and BC be its shadow.
$AB = 30$ m, $BC = 10\sqrt{3}$ m
In ΔABC,



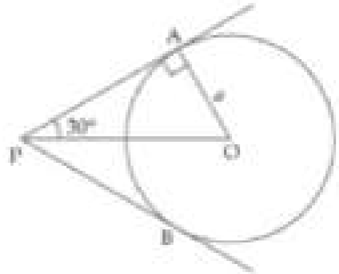
$$\tan\theta = \frac{AB}{BC}$$
$$\Rightarrow \tan\theta = \frac{30}{10\sqrt{3}}$$
$$\Rightarrow \tan\theta = \frac{3}{\sqrt{3}}$$
$$\tan\theta = \sqrt{3}$$

But, $\tan 60° = \sqrt{3}$
$\therefore \theta = 60°$
Thus, the angle of elevation of sun is 60°.

3. In the figure, PA and PB are two tangents from an external point P to a circle with centre O and radius = a

$\angle APB = 60^\circ$ (given)

$\Rightarrow \angle APO = 30^\circ$ (tangents are equally inclined to the line joining the point and the centre)



Now, $OA \perp AP$

In right-angled triangle OAP,

$\sin 30^\circ = \dfrac{OA}{OP}$

$\Rightarrow \dfrac{1}{2} = \dfrac{a}{OP}$

$\Rightarrow OP = 2a$

4. Let a be the first term and d be the common difference of the given A.P.

$\therefore a_{21} - a_7 = 84$

$\Rightarrow (a + 20d) - (a + 6d) = 84$

$\Rightarrow a + 20d - a - 6d = 84$

$\Rightarrow 14d = 84$

$\Rightarrow d = 6$

Hence, the common difference is 6.

SECTION B

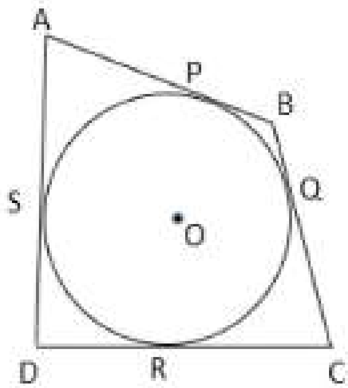
5. Since tangents drawn from an external point to a circle are equal in length, we have

AP = AS ....(i)

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)



Adding (i), (ii), (iii) and (iv), we get

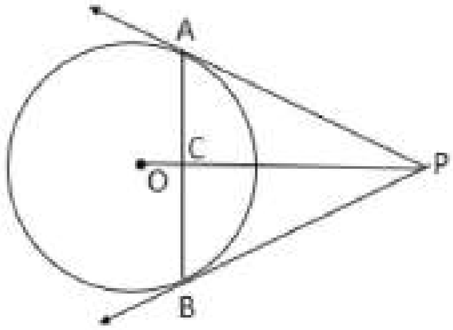
AP + BP + CR + DR = AS + BQ + CQ + DS

$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ AB + CD = BC + DA

6. Let AB be a chord of circle with centre O.
Let AP and BP be two tangents at A and B respectively.
Suppose the tangents meet at point P. Join OP.
Suppose OP meets AB at C.
Now, in ΔPCA and ΔPCB,
PA = PB ....(tangents from an external point are equal)
∠APC = ∠BPC ....(PA and PB are equally inclined to OP)
PC = PC ....(common)
Hence, ΔPAC ≅ ΔPBC ....(by SAS congruence criterion)
⇒ ∠PAC = ∠PBC



7. Since a line is intersecting y-axis at P and x-axis at Q,
Coordinates of P = (0, y) and coordinates of Q = (x, 0)
Let R be the mid-point of PQ.

Then, co-ordinates of R $= \left(\frac{0+x}{2}, \frac{y+0}{2}\right) = (2, -5)$

$\Rightarrow \left(\frac{x}{2}, \frac{y}{2}\right) = (2, -5)$

$\Rightarrow \frac{x}{2} = 2$ and $\frac{y}{2} = -5$

$\Rightarrow x = 4$ and $y = -10$

Hence, co-ordinates of P are $(0, -10)$ and co-ordinates of Q are $(4, 0)$.

8. Given, P(x, y) is equidistant from A(5, 1) and B(−1, 5)
Now, AP = BP

$\Rightarrow \sqrt{(5-x)^2 + (1-y)^2} = \sqrt{(-1-x)^2 + (5-y)^2}$

$\Rightarrow (5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$

$\Rightarrow (25 + x^2 - 10x) + (1 + y^2 - 2y) = (1 + x^2 + 2x) + (25 + y^2 - 10y)$

$\Rightarrow x^2 + y^2 - 10x - 2y + 26 = x^2 + y^2 + 2x - 10y + 26$

$\Rightarrow -10x - 2x = -10y + 2y$

$\Rightarrow -12x = -8y$

$\Rightarrow 3x = 2y$ ....(Dividing throughout by − 4)

9. Given, $px^2 - 14x + 8 = 0$

Here, $a = p$, $b = -14$, $c = 8$

Let α and β be the roots of the given quadratic equation.

Then, $\beta = 6\alpha$

Now, sum of the roots $= \frac{-b}{a}$

$\Rightarrow \alpha + \beta = \frac{-(-14)}{p}$

$\Rightarrow \alpha + \beta = \frac{14}{p}$

$\Rightarrow \alpha + 6\alpha = \frac{14}{p}$

$\Rightarrow 7\alpha = \frac{14}{p}$

$\Rightarrow \alpha = \frac{2}{p} \quad ....(i)$

Product of the roots $= \frac{c}{a}$

$\Rightarrow \alpha\beta = \frac{8}{p}$

$\Rightarrow \alpha \times 6\alpha = \frac{8}{p}$

$\Rightarrow 6\alpha^2 = \frac{8}{p}$

$\Rightarrow 3\alpha^2 = \frac{4}{p}$

$\Rightarrow 3\left(\frac{2}{p}\right)^2 = \frac{4}{p} \quad ....[\text{From (i)}]$

$\Rightarrow 3\left(\frac{4}{p^2}\right) = \frac{4}{p}$

$\Rightarrow p = 3$

10. For A.P. 63, 65, 67, ..., we have

first term = 63 and common difference = 65 − 63 = 2

Hence, n^{th} term $= a_n = 63 + (n - 1)2$

$\Rightarrow a_n = 63 + 2n - 2 = 2n + 61$

For A.P. 3, 10, 17, ..., we have

first term = 3 and common difference = 10 − 3 = 7

Hence, n^{th}term $= a_n^{'} = 3 + (n-1)7$

$\Rightarrow a_n^{'} = 3 + 7n - 7 = 7n - 4$

The two A.Ps will have identical n^{th} term, if

$a_n = a_n^{'}$

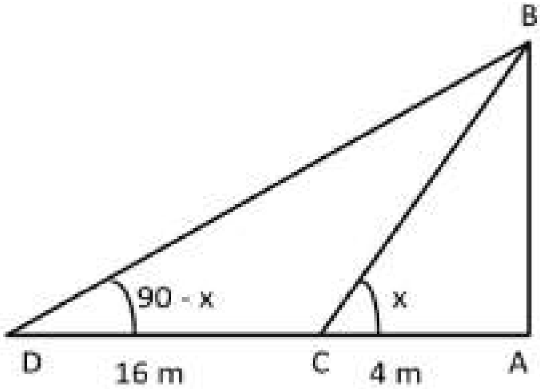
$\Rightarrow 2n + 61 = 7n - 4$

$\Rightarrow 5n = 65$

$\Rightarrow n = 13$

SECTION C

11.



Let AB be the tower with height h.

Let x be the angle of elevation from C.

So, the angle of elevation from D is $(90 - x)$.

....(Since the angles of elevation from C and D are complementary)

In ΔCAB,

$\tan x = \frac{AB}{AC}$

$\Rightarrow \tan x = \frac{h}{4}$ ......(i)

In ΔDAB,

$\tan(90 - x) = \frac{AB}{AD}$

$\Rightarrow \tan(90 - x) = \frac{h}{16}$

$\Rightarrow \cot x = \frac{h}{16}$ ......(ii)

From (i) and (ii),

$\tan x \times \cot x = \frac{h}{4} \times \frac{h}{16}$

$\Rightarrow 1 = \frac{h^2}{64}$

$\Rightarrow h^2 = 64$

$\Rightarrow h = \sqrt{64}$

$\Rightarrow h = 8$ m

Hence, the height of the tower is 8 m.

12. Let the number of black balls in the bag be x.

Number of white balls = 15

Hence, total number of balls in the bag = x + 15

Given, P(black ball) = 3 × P(white ball)

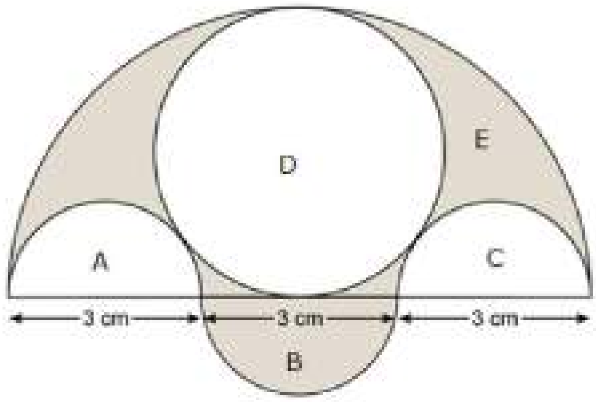
$\Rightarrow \frac{x}{x+15} = 3 \times \frac{15}{x+15}$

$\Rightarrow \frac{x}{x+15} = \frac{45}{x+15}$

$\Rightarrow x = 45$

Thus, the number of black balls in the bag is 45.

13.



Radius of semi-circle A $= \frac{3}{2}$ cm $= 1.5$ cm

Radius of semi-circle B $= \frac{3}{2}$ cm $= 1.5$ cm

Radius of semi-circle C $= \frac{3}{2}$ cm $= 1.5$ cm

Radius of circle D $= \frac{4.5}{2}$ cm $= 2.25$ cm

Radius of semi-circle E = 4.5 cm

Now, area of the shaded region

= Area of semi-circle (E + B) – Area of semi-circle (A + C) – Area of circle D

$= \frac{1}{2}\pi\left[(4.5)^2 + (1.5)^2\right] - \frac{1}{2}\pi\left[(1.5)^2 + (1.5)^2\right] - \pi(2.25)^2$

$= \frac{1}{2}\pi\left[20.25 + 2.25\right] - \frac{1}{2}\pi\left[2.25 + 2.25\right] - \pi(5.0625)$

$= \frac{1}{2}\pi \times 22.50 - \frac{1}{2}\pi \times 4.50 - 5.0625\pi$

$= 11.25\pi - 2.25\pi - 5.0625\pi$

$= 3.9375\pi$

$= 3.9375 \times \frac{22}{7}$

$= 12.375 \text{ cm}^2$

14.

Suppose the point $A\left(\frac{24}{11}, y\right)$ divides the line segment joining points $P(2,-2)$ and $Q(3,7)$ in the ratio $k:1$.

Then, the coordinates of A are $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

But, the coordinates of A are given as $\left(\frac{24}{11}, y\right)$.

$\Rightarrow \frac{3k+2}{k+1} = \frac{24}{11}$

$\Rightarrow 33k + 22 = 24k + 24$

$\Rightarrow 9k = 2 \Rightarrow k = \frac{2}{9}$

Hence, the ratio is $2:9$.

Also, $\frac{7k-2}{k+1} = y \Rightarrow \frac{7 \times \frac{2}{9} - 2}{\frac{2}{9} + 1} = y$

$\Rightarrow y = \frac{\frac{14-18}{9}}{\frac{2+9}{9}} = \frac{-4}{9} \times \frac{9}{11} = -\frac{4}{11}$

15.

We have,

Width of the canal = 5.4m,

Depth of the canal = 1.8 m

It is given that the water is flowing with a speed of 25 km / hr.

Therefore,

Length of the water column formed in 40 mins

that is, $\frac{40}{60}$ hours $= \frac{2}{3}$ hours

is $\frac{2}{3}(25)$ km $= \frac{50}{3}$km $= \frac{50 \times 1000}{3}$ m$= \frac{50000}{3}$ m

$\therefore$ Volume of the water flowing in $\frac{2}{3}$ hours

= Volume of the cuboid of length $\frac{50000}{3}$ m, width 5.4 m and depth 1.8 m

$\Rightarrow$ Volume of the water flowing in $\frac{2}{3}$ hours

$= \frac{50000}{3} \times 5.4 \times 1.8$

$= 162000 \text{ m}^3$

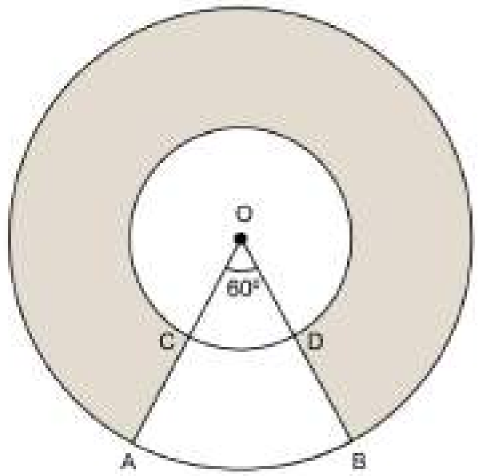
This volume = volume of cuboid (10 cm of standing water is required for irrigation)

This volume = base area of field $\times$ 0.1m

base area $= \frac{162000}{0.1}$

Hence, the cannal irrigates 1620000 m^2 area in 40 mins

16. We have,



Area of the region ABCD

= Area of sector AOB – Area of sector COD

$= \left(\frac{60}{360} \times \frac{22}{7} \times 42 \times 42 - \frac{60}{360} \times \frac{22}{7} \times 21 \times 21\right) \text{cm}^2$

$= \left(\frac{1}{6} \times 22 \times 6 \times 42 - \frac{1}{6} \times 22 \times 3 \times 21\right) \text{cm}^2$

$= (22 \times 42 - 11 \times 21) \text{ cm}^2$

$= (924 - 231) \text{ cm}^2$

$= 693 \text{ cm}^2$

Area of circular ring $= \left(\frac{22}{7} \times 42 \times 42 - \frac{22}{7} \times 21 \times 21\right) \text{cm}^2$

$= (22 \times 6 \times 42 - 22 \times 3 \times 21) \text{ cm}^2$

$= (5544 - 1386) \text{ cm}^2$

$= 4158 \text{ cm}^2$

Hence, Required shaded region = Area of circular ring – Area of region ABCD

$= (4158 - 693) \text{ cm}^2$

$= 3465 \text{ cm}^2$

17. Let the length of the pipe be h cm.

Then, volume of iron pipe = volume of iron in the block.

Volume of the block = $(4.4 \times 2.6 \times 1)m^3 = (440 \times 260 \times 100)\ cm^3$

r = Internal radius of the pipe = 30 cm

R = External radius of the pipe = (30 + 5) cm = 35 cm

∴ Volume of the iron pipe = (External Volume) – (Internal Volume)

$= \pi R^2 h - \pi r^2 h$

$= \pi(R^2 - r^2)h$

$= \pi(R + r)(R - r)h$

$= \pi(35 + 30)(35 - 30)h$

$= \pi \times 65 \times 5 \times h$

Now, Volume of iron in the pipe = Volume of iron in the block

$\Rightarrow \pi \times 65 \times 5 \times h = 440 \times 260 \times 100$

$\Rightarrow \frac{22}{7} \times 65 \times 5 \times h = 440 \times 260 \times 100$

$\Rightarrow h = \frac{440 \times 260 \times 100 \times 7}{22 \times 65 \times 5} = 11200\ cm$

$\Rightarrow h = 112\ m$

Thus, the length of the pipe is 112 m.

18. Radius of common base = 3.5 cm

Total height of toy = 15.5cm

Height of cone = 15.5 – 3.5 = 12cm

For cone,

$l^2 = r^2 + h^2$

$\Rightarrow l^2 = (3.5)^2 + (12)^2$

$\Rightarrow l^2 = 12.25 + 144$

$\Rightarrow l^2 = 156.25$

$\Rightarrow l = \sqrt{156.25} = 12.5cm$

∴ Total surface area of the toy

= Curved surface area of cone + Curved surface area of hemisphere

$= \pi rl + 2\pi r^2$

$= \frac{22}{7} \times 3.5 \times 12.5 + 2 \times \frac{22}{7} \times 3.5 \times 3.5$

$= \frac{22}{7} \times 3.5[12.5 + 7]$

$= \frac{22}{7} \times 3.5 \times 19.5$

$= 214.5\ cm^2$

19. Let there be n terms of this A.P.

For this A.P., $a = 9$

$d = a_2 - a_1 = 17 - 9 = 8$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow 636 = \frac{n}{2}\left[2 \times 9 + (n-1)8\right]$

$\Rightarrow 636 = n\left[9 + (n-1)4\right]$

$\Rightarrow 636 = n\left[9 + 4n - 4\right]$

$\Rightarrow 636 = n(4n + 5)$

$\Rightarrow 4n^2 + 5n - 636 = 0$

$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$

$\Rightarrow n(4n + 53) - 12(4n + 53) = 0$

$\Rightarrow (4n + 53)(n - 12) = 0$

$\Rightarrow 4n + 53 = 0 \text{ or } n - 12 = 0$

$\Rightarrow n = \frac{-53}{4} \text{ or } n = 12$

Since number of terms can neither be negative nor fractional, we have $n = 12$

20. We have

$(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$

The discriminant of the given equation is given by

$D = \left[-2(ac + bd)\right]^2 - 4 \times (a^2 + b^2) \times (c^2 + d^2)$

$\Rightarrow D = 4(ac + bd)^2 - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$\Rightarrow D = 4(a^2c^2 + b^2d^2 + 2abcd) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$\Rightarrow D = 4(a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2)$

$\Rightarrow D = 4(2abcd - a^2d^2 - b^2c^2)$

$\Rightarrow D = -4\left[(ad)^2 + (bc)^2 - 2(ad)(bc)\right]$

$\Rightarrow D = -4(ad - bc)^2$

The given equation will have equal roots, if $D = 0$

$\Rightarrow -4(ad - bc)^2 = 0$

$\Rightarrow (ad - bc)^2 = 0$

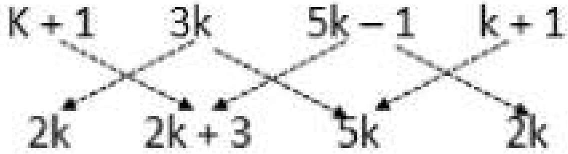
$\Rightarrow ad - bc = 0$

$\Rightarrow ad = bc$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

SECTION D

21. Given points are A(k + 1, 2k), B(3k, 2k + 3) and C(5k − 1, 5k)
These points will be collinear, if area of the triangle formed by them is zero.
We have,



i.e.,

$$\left[k+1 \;\; 2k+3 \; + \; 3k \;\; 5k \; + \; 5k-1 \;\; 2k \right] - \left[3k \;\; 2k \; + \; 5k-1 \;\; 2k+3 \; + \; k+1 \;\; 5k \right] = 0$$

$$\Rightarrow \;\; 2k^2 + 5k + 3 + 15k^2 + 10k^2 - 2k \; - \; 6k^2 + 10k^2 + 13k - 3 + 5k^2 + 5k \;\; = 0$$

$$\Rightarrow \;\; 27k^2 + 3k + 3 \; - \; 21k^2 + 18k - 3 \; = 0$$

$$\Rightarrow 27k^2 + 3k + 3 - 21k^2 - 18k + 3 = 0$$

$$\Rightarrow 6k^2 - 15k + 6 = 0$$

$$\Rightarrow 2k^2 - 5k + 2 = 0$$

$$\Rightarrow 2k^2 - 4k - k + 2 = 0$$

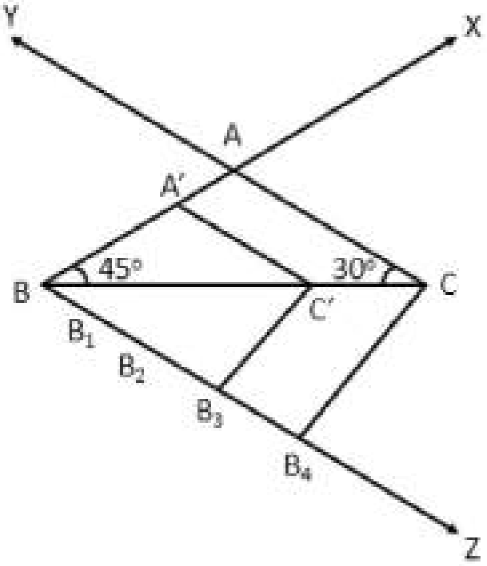
$$\Rightarrow (k-2)(2k-1) = 0$$

$$\Rightarrow k - 2 = 0 \;\; \text{or} \;\; 2k - 1 = 0$$

$$\Rightarrow k = 2 \text{ or } k = \frac{1}{2}$$

22. Steps of construction:
1) Draw BC = 7 cm
2) At B, construct $\angle CBX = 45^\circ$ and at C construct $\angle BCY = 180^\circ - (45^\circ + 105^\circ) = 30^\circ$
3) Let BX and CY intersect at A. ΔABC so obtained is the given triangle.
4) Construct an acute angle $\angle CBZ$ at B on opposite side of vertex A of ΔABC.
5) Mark-off four points (greater of 4 and 3 in 3/4) points B_1, B_2, B_3, B_4 on BZ such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$
6) Join B_4 to C.
7) Draw B_3C' parallel to B_4C which meets BC at C'.
8) From C', draw C'A' parallel to CA meeting BA at A'.

Thus, $\Delta A'BC'$ is the required triangle, each of whose sides is $\frac{3}{4}$ times the corresponding sides of ΔABC.



23. Elementary events associated to the random experiment of throwing two dice are:

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

$\therefore$ Total number of elementary events $= 6 \times 6 = 36$

(i) Let A be the event of getting an even number as the product.

i.e., 2,4,6,8,10,12,

Elementary events favourable to event A are:

(1,1),(1,3),(1,5),(2,2),(2,4),(2,6),(3,1),(3,3),(3,5),
(4,2),(4,4),(4,6),(5,1),(5,3),(5,5),(6,2),(6,4),(6,6)

$\therefore$ Total number of favourable events = 18

Hence, required probability $= \frac{18}{36} = \frac{1}{2}$

(ii) Let B be the event of getting an even number as the sum.

i.e., 2,4,6,8,10,12,16,18,20,24,30,36

Elementary events favourable to event B are:

(1,2),(1,4),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),
(2,6),(3,2),(3,4),(3,6),(4,1),(4,2),(4,3),(4,4),
(4,5),(4,6),(5,2),(5,4),(5,6),(6,1),(6,2),(6,3),
(6,4),(6,5),(6,6)

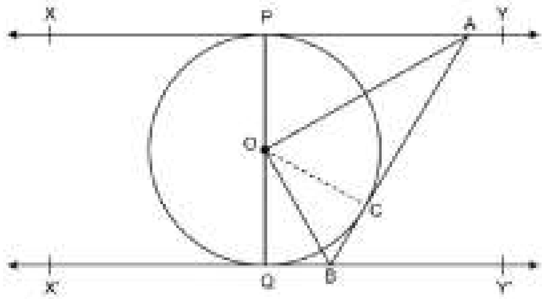
$\therefore$ Total number of favourable events = 27

Hence, required probability $= \frac{27}{36} = \frac{3}{4}$

24. Since tangents drawn from an external point to a circle are equal.

Therefore, AP = AC.

Thus, in triangles AOP and AOC, we have



AP = AC

AO = AO [Common side]

OP = OC [Radii of the same circle]

So, by SSS- criterion of congruence, we have

$\Delta AOP \cong \Delta AOC$

$\Rightarrow \angle PAO = \angle CAO$

$\Rightarrow \angle PAC = 2\angle CAO$

Similarly, we can prove that $\angle QBO = \angle CBO$

$\Rightarrow \angle CBQ = 2\angle CBO$

Now, $\angle PAC + \angle CBQ = 180^0$

[sum of the interior angle on the same side of transversal is 180°]

$\Rightarrow 2\angle CAO + 2\angle CBO = 180°$ [Using equations (i) and (ii)]

$\Rightarrow \angle CAO + \angle CBO = 90°$

$\Rightarrow 180° - \angle AOB = 90°$ [Since $\angle CAO$, $\angle CBO$ and $\angle AOB$ are angles of a Triangle, $\angle CAO + \angle CBO + \angle AOB = 180°$]

$\Rightarrow \angle AOB = 90°$

25. We have,

r = Radius of cylindrical vessel = 1m

h = Height of cylindrical vessel = 3.5 m

$\therefore$ Volume of cylindrical vessel $= \pi r^2 h = \frac{22}{7} \times 1^2 \times 3.5\ m^3 = 11\ m^3$

Let the rainfall be x m.

Then, Volume of the water

= Volume of cuboid of base 22 m $\times$ 20 m and height x metres

$= (22 \times 20 \times x)\ m^3$

Since the vessel is just full of the water that drains out of the roof into the vessel,

Volume of the water = Volumeofthecylindricalvessel

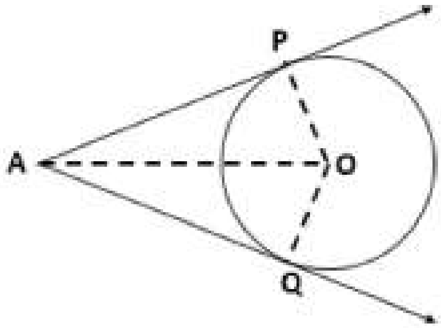
$\Rightarrow 22 \times 20 \times x = 11$

$\Rightarrow x = \frac{11}{22 \times 20} = \frac{1}{40}\ m = \frac{100}{40}\ cm = 2.5\ cm$

Thus, the rainfall is 2.5 cm.

26. Given: AP and AQ are two tangents from a point A to a circle C(O, r)
To prove: AP = AQ
Construction: Join OP, OQ and OA

Proof:



In ΔOPA and ΔOQA,

$\angle OPA = \angle OQA = 90^{\circ}$ ....(Tangent at any point of a circle is perpendicular to the radius through the point of contact)

$OP = OQ$ ....(Radii of a circle)

$OA = OA$ ....(Common)

Hence, by RHS-criterion of congruence, we have

$\Delta OPA \equiv \Delta OQA$

$\Rightarrow AP = AQ$ ....(c.p.c.t)

27. Let a_1, a_2, be the first terms and d_1, d_2 the common differences of the two given A.P's.

Then, sum of their n terms is given by

$S_n = \frac{n}{2}\left[2a_1 + (n-1)d_1\right]$ and $S_n' = \frac{n}{2}\left[2a_2 + (n-1)d_2\right]$

$$\therefore \frac{S_n}{S_n'} = \frac{\frac{n}{2}\left[2a_1 + (n-1)d_1\right]}{\frac{n}{2}\left[2a_2 + (n-1)d_2\right]} = \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2}$$

It is given that,

$$\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27} \quad ....(i)$$

In order to find the ratio of the m^{th} terms of the two given A.P's, we replace n by $\left[2m-1\right]$ in equation (i).

Thus, to find the ratio of the 9^{th} terms of the two given A.P's, we replace n by 17 $\left[2 \times 9 - 1\right]$ in equation (i)

$$\frac{2a_1 + (17-1)d_1}{2a_2 + (17-1)d_2} = \frac{7 \times 17 + 1}{4 \times 17 + 27}$$

$$\Rightarrow \frac{2a_1 + 16d_1}{2a_2 + 16d_2} = \frac{120}{95}$$

$$\Rightarrow \frac{a_1 + 8d_1}{a_2 + 8d_2} = \frac{24}{19}$$

Thus, the ratio of their 9^{th} terms is 24:19.

28. $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2$

$\frac{x-1^{2}+\ 2x+1^{2}}{2x+1\ \ x-1}=2$

$\Rightarrow \frac{x^2-2x+1+4x^2+4x+1}{2x^2-2x+x-1}=2$

$\Rightarrow \frac{5x^2+2x+2}{2x^2-x-1}=2$

$\Rightarrow 5x^2+2x+2=4x^2-2x-2$

$\Rightarrow 5x^2+2x+2-4x^2+2x+2=0$

$\Rightarrow x^2+4x+4=0$

$\Rightarrow x^2+2x+2x+4=0$

$\Rightarrow x\ \ x+2\ \ +2\ \ x+2\ \ =0$

$\Rightarrow \ x+2\ \ x+2\ \ =0$

$\Rightarrow \ x+2^{\ 2}=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

29. Suppose B alone takes x days to finish the work.
Then, A alone can finish it in (x – 6) days
Now,

(A's one day's work) + (B's one day's work) = $\frac{1}{x-6}+\frac{1}{x}$

And, (A + B)'s one day's work = $\frac{1}{4}$

$\therefore \frac{1}{x-6}+\frac{1}{x}=\frac{1}{4}$

$\Rightarrow \frac{x+x-6}{x(x-6)}=\frac{1}{4}$

$\Rightarrow \frac{2x-6}{x^2-6x}=\frac{1}{4}$

$\Rightarrow 8x-24=x^2-6x$

$\Rightarrow x^2-6x-8x+24=0$

$\Rightarrow x^2-14x+24=0$

$\Rightarrow x^2-12x-2x+24=0$

$\Rightarrow x(x-12)-2(x-12)=0$

$\Rightarrow (x-12)(x-2)=0$

$\Rightarrow x-12=0$ or $x-2=0$

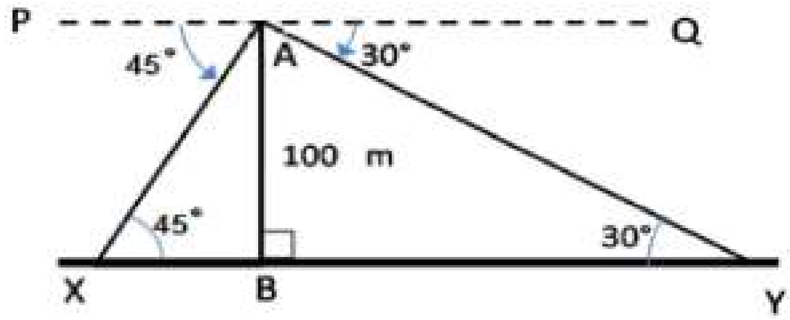
$\Rightarrow x=12$ or $x=2$

But, x cannot be less than 6.

So, $x=12$

Hence, B alone can finish the work in 12 days.

30.



The man is at the top of the tower AB.

In right angled triangle ΔABX and ΔABY,

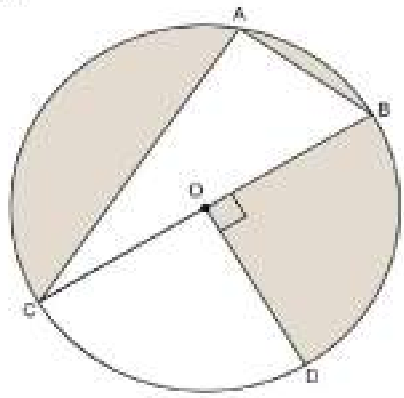
$\tan 45^\circ = \frac{AB}{XB} \Rightarrow 1 = \frac{100}{XB} \Rightarrow XB = 100\,m$

$\tan 30^\circ = \frac{AB}{YB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{YB} \Rightarrow YB = 100\sqrt{3}$

$XY = XB + YB = 100 + 100\sqrt{3} = 273.20$ (approx)

Hence, the distance between X and Y is 273.20 m approximately.

31. AC = 24 cm, AB = 7 cm



Since BC is the diameter of the circle,

so, $\angle BAC = 90^\circ$

In right ΔBAC,

$BC^2 = AC^2 + AB^2$

$\Rightarrow BC^2 = 24^2 + 7^2$

$\Rightarrow BC^2 = 625$

$\Rightarrow BC = 25$ cm

So, the radius of the circle = OC = 12.5 cm

Area of the shaded region

= Area of the circle – Area of ΔBAC – Area of sector CD

$= \pi r^2 - \frac{1}{2} \times AB \times AC - \frac{\theta}{360} \times \pi r^2$

$= \left(\frac{22}{7} \times 12.5 \times 12.5\right) - \left(\frac{1}{2} \times 7 \times 24\right) - \left(\frac{90}{360} \times \frac{22}{7} \times 12.5 \times 12.5\right)$

....($\because \angle BOD = 90^\circ \Rightarrow \angle COD = 90^\circ$)

$= 491.07 - 84 - 122.77$

$= 284.3\ cm^2$ (approximately)

Hence, the area of the shaded region is $284.3\ cm^2$ approximately.