CBSE Board
Class X – Set 3
Mathematics
Board Question Paper 2018

Time: 3 hrs. **Marks: 80**

Note:

- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **30** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions:

(i) **All** questions are compulsory.

(ii) This question paper consists of **30** questions divided into four sections - A, B, C and D.

(iii) Section A contains **6** questions of **1** mark each. Section B contains **6** questions of marks cock Section C contains **10** questions of **3** marks each. Section D contains **8** questions of **4** marks each.

(iv) There is no overall choice. However, an internal choice has been provided in four questions of **3** marks each and **3** questions of **4** marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculator is not permitted.

SECTION A

Question numbers **1** to **6** carry **1** mark each.

1. What is the value of $(\cos^2 67^\circ - \sin^2 23^\circ)$?

2. In an AP. if the common difference (d) = -4, and the seventh term (a_7) is 4, then find the first term.

3. Given $\Delta\, ABC - \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{ar\Delta ABC}{ar\Delta PQR}$.

4. What is the HCF of smallest prime number and the smallest composite number?

5. Find the distance of a point P(x, y) from the origin.

6. if x = 3 is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.

SECTION B

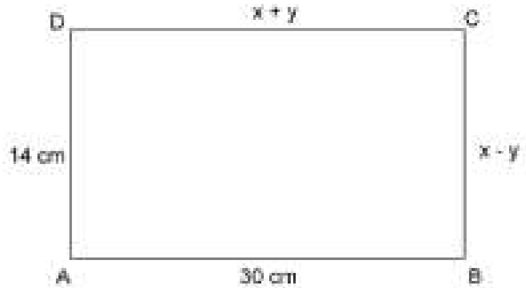
Question numbers **7** to **12** carry **2** marks each.

7. Two different dice are tossed together. Find the probability:
(i) of getting a doublet
(ii) of getting a sum 10, of the numbers on the two dice.

8. Find the radio in which P (4, m) divides the line segment joining the points A (2, 3) and B (6, -3). Hence find m.

9. An integer is chosen at random between 1 and 100. Find the probability that it is:
(i) divisible by 8.
(ii) not divisible by 8.

10. In Fig.1 ABCD is a rectangle. Find the values of x and y.



11. Find the sum of first 8 multiples of 3.

12. Given that $\sqrt{2}$ is irrational, prove that $\left(5+3\sqrt{2}\right)$ is an irrational number.

SECTION – C

Question numbers **13** to **22** carry **3** marks each.

13. If A(-2, 1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.

OR

If A(-5, 7), B(-4, -5), C(-1, -6) and D(4, 5) are the vertices of a quadrilateral, find the area of quadrilateral ABCD.

14. Find all zeros of the polynomial $(2x^4-9x^3+5x^2+3x-1)$ if two of its zeros are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

15. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.

16. Prove that the lengths of tangents drawn from an external point to a circle are equal.

17. Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal.

OR

If the area of two similar triangles are equal, prove that they are congruent

18. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

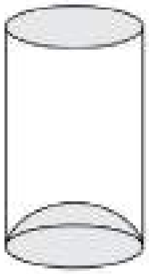
19.

The table below shows the salaries of 280 persons :

Salary (In thousand)	No. of persons
5-10	49
10-15	133
15-20	63
20-25	15
25-30	6
30-35	7
35-40	4
40-45	2
45-50	1

Calculate the median salary of the data.

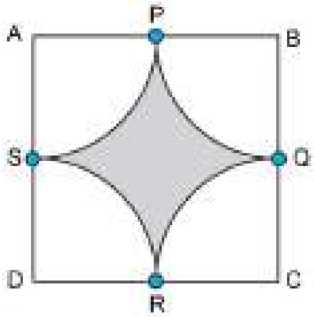
20. A wooden article was made by scooping out of hemisphere from each end of a solid cylinder, as shown in Fig.2. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. find the total surface area of the article.



OR

A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

21. Find the area of the shaded region in Fig.3, where areas drawn with centres A,B,C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD of side 12 cm (Use $\pi = 3.14$)



22. if $4 \tan \theta = 3$, evaluate $\left(\frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta - \cos \theta - 1}\right)$

OR

If $\tan 2A = \cot (A - 18^{\circ})$, where 2A is an acute angle, find the value of A.

SECTION – D

Question numbers **23** to **30** carry **4** marks each.

23. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. (Use $\sqrt{3}=1.732$)

24. The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find:
(i) The area of the metal sheet used to make the bucket.
(ii) Why we should avoid the bucket made by ordinary plastic? (Use $\pi = 3.14$)

25. Prove that : $\frac{\sin A - 2 \sin^3 A}{2 \cos^3 A - \cos A} = \tan A.$

26. The mean of the following distribution is 18. Find the frequency f of the class 19-21.

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

OR

The following distribution gives the daily income of 50 workers of a factory:

Daily Income (In)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type of cumulative frequency distribution and draw its ogive.

27. A motor boat whose speed is 18km/hr in still water takes 1hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

OR

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed?

28. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7:15. Find the numbers.

29. Draw a triangle ABC with BC = 6 cm AB = 5 cm and $\angle$ ABC = 60° Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the ΔABC.

30. In an equilateral ΔABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that

$9(AD)^2 = 7(AB)^2$

OR

Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

CBSE
Class X Mathematics
Board Paper – 2018 Solution
All India Set – 3

Time: 3 hours **Total Marks: 80**

SECTION A

1.

$$\cos^2 67° - \sin^2 23°$$
$$= \cos^2(90° - 23°) - \sin^2 23° \qquad (\because \cos^2\theta = \sin^2(90° - \theta))$$
$$= \sin^2 23° - \sin^2 23°$$
$$= 0$$

2. Given that d = −4, $a_7 = 4$, n = 7

$$a_7 = 4$$
$$\text{Now, } a_n = a + (n-1)d$$
$$\therefore a + (7-1)(-4) = 4$$
$$\therefore a - 6 \times 4 = 4$$
$$\therefore a - 24 = 4$$
$$\therefore a = 28$$

$\therefore$ First term of the given A.P. is 28.

3. Given that $\Delta ABC \sim \Delta PQR$

$$\frac{AB}{PQ} = \frac{1}{3}$$

Using theorem "If two triangles are similar then the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$$
$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{1}{3}\right)^2$$
$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{1}{9}$$

4. Smallest prime number is 2 and smallest composite number is 4.

$2 = \underline{2} \times 1$

$4 = \underline{2} \times 2$

$\therefore$ HCF of 2 and 4 is 2.

5. According to the question,

origin O (0, 0) and point P (x, y)

Distance of a point P(x, y) from O(0, 0) = $OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$ units

6. Let $p(x) = x^2 - 2kx - 6$

x = 3 is one root of given quadratic equation.

$\therefore p(3) = 0$

$\therefore 3^2 - 2k \times 3 - 6 = 0$

$\therefore 9 - 6k - 6 = 0$

$\therefore 3 = 6k$

$\therefore k = \frac{1}{2}$

SECTION B

7. When two dice are tossed together, the sample space S is given by

$$S = \left\{ \begin{array}{l} (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), \\ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), \\ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), \\ (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), \\ (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \end{array} \right\}$$

$\therefore n(S) = 36$

(i) Let A be the event of gettinga doublet.

$\therefore A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$\therefore n(A) = 6$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii) Let B be the event of getting a sum of 10 of the numbers on two dice.

$\therefore B = \{(5,5), (6,4), (4,6)\}$

$\therefore n(B) = 3$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

8. Let P(4, m) divides the line segment joining the points A(2, 3) and B(6, –3) in the ratio t : 1.

Co – ordinates of point P are $\left(\frac{6t+2}{t+1}, \frac{-3t+3}{t+1}\right)$.

Given co – ordinates of point P are $(4, m)$.

$\therefore \left(\frac{6t+2}{t+1}, \frac{-3t+3}{t+1}\right) = (4, m)$

$\therefore \frac{6t+2}{t+1} = 4$ and $\frac{-3t+3}{t+1} = m$

Consider $\frac{6t+2}{t+1} = 4$

$\therefore 6t + 2 = 4t + 4$

$\therefore 2t = 2$

$\therefore t = 1$

Put $t = 1$ in $\frac{-3t+3}{t+1} = m$

$\therefore \frac{-3 \times 1 + 3}{1+1} = m$

$\therefore m = 0$

9. According to the question sample space S is as follows:

$S = \{2, 3, 4, 5, \ldots\ldots, 99\}$

$\therefore n(S) = 98$

(i) Let A be the event that chosen integer is divisible by 8.

$\therefore A = \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96\}$

$\therefore n(A) = 12$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{12}{98} = \frac{6}{49}$

Thus, probability that a chosen integer is divisible by 8 is $\frac{6}{49}$.

(ii) Probability that a chosen integer is not divisible by 8 $= P(A') = 1 - P(A) = \frac{43}{49}$.

10. According to the figure,

$x + y = 30$ ....(i)

$x - y = 14$ ....(ii)

Adding (i) and (ii),

$\therefore 2x = 44$

$\therefore x = 22$

Put $x = 22$ in (i)

$\therefore 22 + y = 30$

$\therefore y = 30 - 22$

$\therefore y = 8$

$\therefore$ The values of x and y are 22 and 8 respectively.

11. First 8 multiples of 3 are {3, 6, 9, 12, 15, 18, 21, 24}.

This forms an A.P. with

first term, $a = 3$,

common difference, $d = 6 - 3 = 3$,

last term, $l = 24$

and $n = 8$

$S_n = \frac{n}{2}(a + l)$

$\therefore S_8 = \frac{8}{2}(3 + 24)$

$\therefore S_8 = 4 \times 27$

$\therefore S_8 = 108$

$\therefore$ Sum of first 8 multiples of 3 is 108.

12. Let us assume that $5 + 3\sqrt{2}$ is a rational number.

Hence, we can find co – prime p and q $(q \neq 0)$ such that

$$5+3\sqrt{2}=\frac{p}{q}$$

$$\therefore \frac{p}{q}-5=3\sqrt{2}$$

$$\therefore \frac{p}{3q}-\frac{5}{3}=\sqrt{2}$$

$$\therefore \frac{p-5q}{3q}=\sqrt{2}$$

But $\sqrt{2}$ is irrational.

Thus, our assumption is wrong.

Hence, $5+3\sqrt{2}$ is an irrational.

SECTION C

13.

Diagonals of parallelogam bisect each other

so midpoint of AC = mid point of BD

$$\therefore \left(\frac{4-2}{2},\frac{b+1}{2}\right)=\left(\frac{a+1}{2},\frac{0+2}{2}\right)$$

$$\therefore \left(\frac{2}{2},\frac{b+1}{2}\right)=\left(\frac{a+1}{2},\frac{2}{2}\right)$$

$$\therefore \left(1,\frac{b+1}{2}\right)=\left(\frac{a+1}{2},1\right)$$

After comparing the coordinates, we get

$$1=\frac{a+1}{2} \quad \text{and} \quad \frac{b+1}{2}=1$$

$$\therefore a+1=2 \quad \text{and} \quad b+1=2$$

$$\therefore a=1 \quad \text{and} \quad b=1$$

$\therefore$ the coordinates are A(-2, 1), B(1, 0), C(4, 1), and D(1, 2) Now using distance formula,

As we know, Diagonals of a Parallelogram are equal in length.

$$\therefore AB = DC = \sqrt{(1+2)^2+(0-1)^2}=\sqrt{9+1}=\sqrt{10} \quad \text{(By distance formula)}$$

$$\therefore AD = BC = \sqrt{(1+2)^2+(2-1)^2}=\sqrt{9+1}=\sqrt{10}$$

OR

We will join B and D so that we will be getting two triangles ABD and BCD.
Then we will find their inividual areas and add them together
to find the Area of a Quadrialteral ABCD.

$$\text{Area of } \Delta ABD = \frac{1}{2}\left[-5(-5-5)-4(5-7)+4(7+5)\right] \text{............by formula(Coordinate Geometry)}$$

$$=\frac{1}{2}(50+8+48)$$

$$=\frac{106}{2}$$

$$=53 \text{ sq.units}$$

$$\text{Area of } \Delta BCD = \frac{1}{2}\left[-4(-6-5)-1(5+5)+4(-5+6)\right] \text{............by formula(Coordinate Geometry)}$$

$$=\frac{1}{2}(44-10+4)$$

$$=\frac{38}{2}$$

$$=19 \text{ sq.units}$$

$\therefore$ Area of $\triangle$ABCD = Area of ΔABD + Area of ΔBCD

$$=53+19$$

$$=72 \text{ sq.units}$$

14.

Now $2+\sqrt{3}$ and $2-\sqrt{3}$ are the two zeroes of the given polynomial

So the product $\left[x-\left(2+\sqrt{3}\right)\right]\left[x-\left(2-\sqrt{3}\right)\right]$ will be a factor of the given polynomial

$$\therefore \left[x-\left(2+\sqrt{3}\right)\right]\left[x-\left(2-\sqrt{3}\right)\right]=(x-2)^2-\left(\sqrt{3}\right)^2$$

$$=x^2-4x+4-3$$

$$=x^2-4x+1$$

let $f(x)=2x^4-9x^3+5x^2+3x-1$

and $g(x)=x^2-4x+1$

Find $\frac{f(x)}{g(x)}$.

$$\begin{array}{r|l} & 2x^2-x-1 \\ x^2-4x+1 & 2x^4-9x^3+5x^2+3x-1 \\ & 2x^4-8x^3+2x^2 \\ & -\quad+\quad- \\ \hline & -x^3+3x^2+3x \\ & -x^3+4x^2-x \\ & +\quad-\quad+ \\ \hline & -x^2+4x-1 \\ & -x^2+4x-1 \\ & +\quad-\quad+ \\ \hline & 0 \end{array}$$

$\therefore f(x)=\left(x^2-4x+1\right)\left(2x^2-x-1\right)$

$\therefore 2x^4-9x^3+5x^2+3x-1=\left(x^2-4x+1\right)\left(2x^2-x-1\right)$

Hence, the other zeroes of f(x) are the zeroes of the Polynomial $2x^2-x-1$.

$\therefore 2x^2-x-1=2x^2-2x+x-1=(2x+1)(x-1)$

So, $2x^4-9x^3+5x^2+3x-1=\left(x^2-4x+1\right)\left(2x^2-x-1\right)$

$$=\left[x-\left(2+\sqrt{3}\right)\right]\left[x-\left(2-\sqrt{3}\right)\right](2x+1)(x-1)$$

Hence the roots of the Polynomial f(x) are $\left(2+\sqrt{3}\right),\left(2-\sqrt{3}\right),\frac{-1}{2}$ and 1.

15.

We will find the Prime factors of 404 and 96.

$\therefore 404 = 2\times2\times101$

$\therefore 96 \;= 2\times2\times2\times2\times2\times3$

So,

$\text{LCM}(404,96) = 2\times2\times2\times2\times2\times3\times101 = 9696$

$\text{HCF} = 2\times2 = 4$

Here,

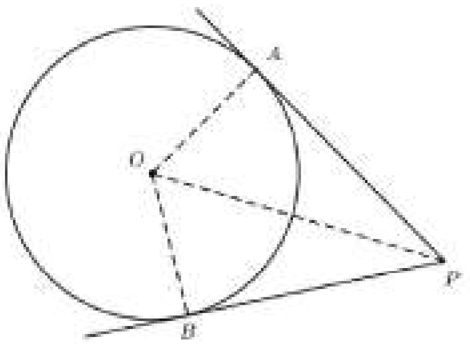
$\text{LCM}\times\text{HCF} = 9696\times4 = 38784$.............i)

Product of numbers $= 404\times96 = 38784$............ii)

$\therefore$ LCM $\times$ HCF = Product of numbers (from i and ii)

Hence verified.

16.



Here,

PA and PB are tangents to the circle with centre O,

and AO and OB are the radii of the Circle.

$\therefore \left.\begin{matrix} PA \perp AO \\ PB \perp BO \end{matrix}\right\}$.....tangent $\perp$ to radius

In ΔOPA and ΔOPB

$\angle OAP = \angle OBP$ .....each 90° (radius and tangent are $\perp$ at their poit of contact)

$OA = OB$ .....(radii of the same circle)

$OC = OC$ .....(common)

$\Delta OPA \cong \Delta OPB$........(by RHS Theorem)

$\therefore PA = PB$......(CPCT)

Hence Proved

17.

Let sides of a square be x units

$\therefore$ Diagonal length = $\sqrt{2}x$ units

Since, Area of an Equilateral triangle $=\frac{\sqrt{3}}{4}(\text{side})^2$

$\therefore$ Area of an Equilateral triangle described on side $=\frac{\sqrt{3}}{4}(x)^2$ ....i)

$\therefore$ Area of equilateral triangle described on diagonal $=\frac{\sqrt{3}}{4}\left(\sqrt{2}x\right)^2$ ....ii)

$$\therefore \text{Ratio of their areas} = \frac{\text{Area of equilateral triangle described on side}}{\text{Area of equilateral triangle described on diagonal}}$$

$$= \frac{\frac{\sqrt{3}}{4}(x)^2}{\frac{\sqrt{3}}{4}\left(\sqrt{2}x\right)^2} \text{.............(from i and ii)}$$

$$= \frac{1}{2}$$

$$\because \frac{\text{Area of equilateral triangle described on side}}{\text{Area of equilateral triangle described on diagonal}} = \frac{1}{2}$$

$\therefore$ Area of equilateral triangle on side $=\frac{1}{2}\times$ Area of equilateral triangle on diagonal

OR

Suppose two similar triangles are ΔABC and ΔPQR.

Given,

Area of ΔABC = Area of ΔPQR

Now using properties of similar triangles, we get

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \text{......(CPCT)..........i)}$$

Also, we know that ratio of areas of two Similar Triangles is equal to the ratio of their corresponding sides

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AB^2}{PQ^2}$$

$$\therefore 1 = \frac{AB^2}{PQ^2}$$

$$\therefore 1 = \frac{AB}{PQ}$$

$\therefore AB = PQ$

so, from i)

$$\frac{AC}{PR} = \frac{BC}{QR} = 1$$

$\therefore AC = PR$

$\therefore BC = PQ$

$\Delta ABC \cong \Delta PQR$.....(SSS test)

18.

Here,

Distance $=1500\text{km}$

As we know,

$$\text{Time}=\frac{\text{Distance}}{\text{Speed}}$$

Let the usual speed of plane be x km/hr.

$$\therefore T_1=\frac{1500}{x}\text{hr}$$

After increasing the speed by 100km/hr it's speed becomes $(x+100)$ km/hr.

$$\therefore T_2=\frac{1500}{x+100}\text{hr}$$

Given that difference in speed is 30 mins which is $\frac{1}{2}$ hours

$$\therefore T_1-T_2=\frac{1}{2}$$

$$\therefore \frac{1500}{x}-\frac{1500}{x+100}=\frac{1}{2}$$

$$\therefore \frac{1500(x+100)-1500x}{x(x+100)}=\frac{1}{2}$$

$$\therefore x^2+100x-300000=0$$

$$\therefore x^2+600x-500x-300000=0$$

$$\therefore (x+600)(x-500)=0$$

$$\therefore x=-600 \text{ or } x=500$$

Speed can't be negative

$$\therefore x=500 \text{ km/hr}$$

Therefore, the usual speed of plane is 500 km/hr.

19.

Salary	Frequency	CF
5-10	49	49
10-15	133	182
15-20	63	245
20-25	15	260
25-30	6	266
30-35	7	273
35-40	4	277
40-45	2	279
45-50	1	280

$\therefore \frac{N}{2} = \frac{280}{2} = 140$

The cumulative frequency which is greater and nearest to 140 is 182.

So,

Median Class is 10-15

Thus,

l = 10

h = 5

N = 280

cf = 49

f = 133

Median for the grouped date is given by,

$$\therefore \text{Median} = l + \left[\frac{\left(\frac{N}{2} - cf\right)}{f}\right] \times h$$

$$= 10 + \left[\frac{(140 - 49)}{133}\right] \times 5$$

$$= 13.42$$

20.

Radius of cylindrical part = Radius of Hemisperical Part = 3.5 cm

Height of cylindrical part = h =10 cm

Here,

Surface Area of the article = Curved surface Area of Cylinder + 2 × Curved surface Area of Hemisphe

$$=2\pi rh + 2\times 2\pi r^2$$
$$=2\pi\times 3.5\times 10 + 2\times 2\pi\times 3.5\times 3.5$$
$$=70\pi+49\pi$$
$$=119\times\pi$$
$$=374\ cm^2$$

OR

Volume of rice (heap) = volume of cone = $\frac{1}{3}\pi r^2h$

$r=12m,\ h=3.5m$

$$\text{Volume} = \frac{1}{3}\pi r^2h=\frac{1}{3}\pi\times12\times12\times3.5$$
$$=\frac{1}{3}\times\frac{22}{7}\times12\times12\times3.5$$
$$=528m^3$$

Area of canvas cloth = curved surface area of the cone = πrl

$\therefore l=\sqrt{r^2+h^2}=\sqrt{12^2+3.5^2}=12.5\ m$

Area of canvas cloth $=\pi rl=\frac{22}{7}\times12\times12.5=471.42m^2$

Therefore Area of canvas required to cover the heap of the rice is $471.42m^2$

21.

According to the question,

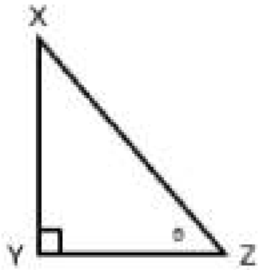
Here side of the square = 12 cm and r = $\frac{\text{side}}{2}$ = 6cm and $\theta = 90^\circ$ (angle of the square)

$\therefore$ Area of shaded region = Area of the square ABCD − 4(Area of quadrant)

$$=\text{side}^2-4\times\left(\frac{\theta}{360^\circ}\times\pi r^2\right)$$
$$=12^2-4\times\left(\frac{90^\circ}{360^\circ}\times3.14\times6^2\right)\quad\left(\because r=\frac{1}{2}a\right)$$
$$=144-113.04$$
$$=30.96\ cm^2$$

$\therefore$ Area of shaded region is $30.96\ cm^2$.

22.



Let ΔXYZ is a right angle triangle at Y and $\angle Z = \theta$.

given, $4\tan\theta = 3$

$$\therefore \tan\theta = \frac{3}{4}$$

Let $XY = 3k$ and $YZ = 4k$

By Pythagoras Theorem, we get

$\therefore XZ^2 = XY^2 + YZ^2$

$\therefore XZ^2 = (3k)^2 + (4k)^2$

$\therefore XZ^2 = 9k^2 + 16k^2$

$\therefore XZ^2 = 25k^2$

$\therefore XZ = 5k$

$\therefore \sin\theta = \frac{XY}{XZ} = \frac{3k}{5k} = \frac{3}{5}$ and $\cos\theta = \frac{YZ}{XZ} = \frac{4k}{5k} = \frac{4}{5}$

Substituting the values in $\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}$, we get

$$\frac{4\times\frac{3}{5} - \frac{4}{5} + 1}{4\times\frac{3}{5} + \frac{4}{5} - 1} = \frac{\frac{12-4+5}{5}}{\frac{12+4-5}{5}}$$

$$= \frac{13}{11}$$

$$\therefore \frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{13}{11}$$

OR

According to the question,

$\tan 2A = \cot(A - 18^\circ)$

$\therefore \tan 2A = \tan[90^\circ - (A - 18^\circ)]$ $\quad [\because \cot A = \tan(90^\circ - A)]$

$\therefore \tan 2A = \tan(90^\circ - A + 18^\circ)$

$\therefore 2A = 90^\circ - A + 18^\circ$

$\therefore 2A + A = 108^\circ$

$\therefore 3A = 108^\circ$

$\therefore A = 36^\circ$

SECTION D

23. Let A and B be the two ships.

Let d be the distance between the two ships.

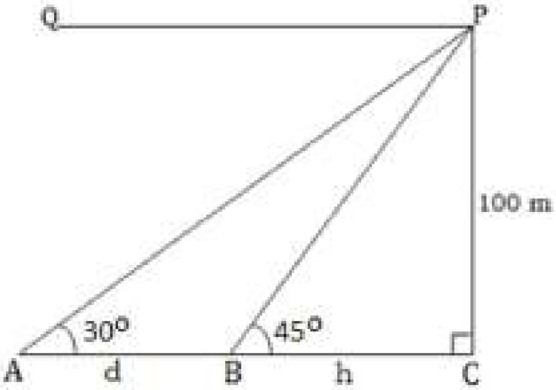
i.e. AB = d metres.

Suppose that the observer is at the point P.

It is given that PC = 100 m.

Let h be the distance (in metres) from B to C.

In right ΔPCA,



$$\cot 30^\circ = \frac{AC}{PC}$$
$$\Rightarrow \sqrt{3} = \frac{d+h}{100}$$
$$\Rightarrow d + h = 100\sqrt{3} \quad ....(1)$$

In right ΔPCB,

$$\cot 45^\circ = \frac{BC}{PC}$$
$$\Rightarrow 1 = \frac{h}{100}$$
$$\Rightarrow h = 100 \text{ m} \quad ....(2)$$

Substituting the value of h in (1), we get

$$d + 100 = 100\sqrt{3}$$
$$\Rightarrow d = 100\sqrt{3} - 100$$
$$\Rightarrow d = 100\left(\sqrt{3} - 1\right) = 100 \times 0.732 = 73.2 \text{ m}$$

Thus, the distance between two ships is 73.2 m.

24. Height of the cone, h = 24 cm

Upper radius of the cone, R = 15 cm

Lower radius of the cone, r = 5cm

Slant height of the cone,

$l = \sqrt{h^2 + (R-r)^2} = \sqrt{24^2 + (15-5)^2} = \sqrt{576+100} = \sqrt{676} = 26$ cm

Now, area of metal sheet used to make the bucket

= Total surface area of the bucket

$= \pi(R+r)l + \pi r^2$

$= \pi(15+5)\times 26 + \pi \times (5)^2$

$= \pi[520+25]$

$= \pi \times 545$

$= 3.14 \times 545$

$= 1711.3\ cm^2$

Plastics are not biodegradable. That is, plastic material mostly end as harmful waste that pollutes the environment and causes health problems, we should avoid using plastic.

25. $L.H.S. = \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$

$= \frac{\sin A(1-\sin^2 A - \sin^2 A)}{\cos A[2\cos^2 A - (\sin^2 A + \cos^2 A)]}$

$= \frac{\sin A(\cos^2 A - \sin^2 A)}{\cos A(\cos^2 A - \sin^2 A)}$

$= \frac{\sin A}{\cos A}$

$= \tan A$

$=$ R.H.S.

26. We have,

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$
$11-13$	3	12	36
$13-15$	6	14	84
$15-17$	9	16	144
$17-19$	13	18	234
$19-21$	f	20	20f
$21-23$	5	22	110
$23-25$	4	24	96
	$\sum f_i = 40+f$		$\sum f_i x_i = 704+20f$

Now, Mean $= \frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 18 = \frac{704+20f}{40+f}$

$\Rightarrow 18(40+f) = 704+20f$

$\Rightarrow 720+18f = 704+20f$

$\Rightarrow 2f = 16$

$\Rightarrow f = 8$

Thus, the missing frequency is 8.

OR

Less Than Series:

Class interval	Cumulative Frequency
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

Now, we mark the upper class limits along X-axis and the cumulative frequencies along Y-axis.

Thus, we plot the points (120, 12), (140, 26), (160, 34), (180, 40), (200, 50) to get 'less than type' ogive.

27. Let x be the speed of the stream.

$\therefore$ Speed of the boat while going upstream $= (18 - x)$ km / hr

And, speed of the boat while going downstream $= (18 + x)$ km / hr

Time taken by the boat to go upstream $= \frac{24}{18-x}$ hours

Time taken by the boat to go downstream $= \frac{24}{18+x}$ hours

It is given that time taken for going upstream is 1 hour more than the time taken for going downstream.

Thus, we have

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\Rightarrow 24\left[\frac{1}{18-x} - \frac{1}{18+x}\right] = 1$$

$$\Rightarrow \frac{18+x-18+x}{(18-x)(18+x)} = \frac{1}{24}$$

$$\Rightarrow \frac{2x}{324-x^2} = \frac{1}{24}$$

$$\Rightarrow 48x = 324 - x^2$$

$$\Rightarrow x^2 + 48x - 324 = 0$$

$$\Rightarrow x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow x(x+54) - 6(x+54) = 0$$

$$\Rightarrow (x-6)(x+54) = 0$$

$$\Rightarrow x - 6 = 0 \quad \text{or} \quad x + 54 = 0$$

$$\Rightarrow x = 6 \qquad \text{or} \quad x = -54$$

As speed cannot be negative, the speed of the stream is 6 km/hr.

OR

Let the original average speed of the train be x km/hr.

Time taken by train to cover a distance of 63 km = $\frac{63}{x}$ hours

Time taken by train to cover a distance of 72 km = $\frac{72}{x+6}$ hours

According to given condition,

$\frac{63}{x}+\frac{72}{x+6}=3$

$\Rightarrow \frac{63(x+6)+72x}{x(x+6)}=3$

$\Rightarrow 63x+378+72x=3x^2+18x$

$\Rightarrow 135x+378=3x^2+18x$

$\Rightarrow 3x^2-117x-378=0$

$\Rightarrow x^2-39x-126=0$

$\Rightarrow x^2-42x+3x-126=0$

$\Rightarrow x(x-42)+3(x-42)=0$

$\Rightarrow (x-42)(x+3)=0$

$\Rightarrow x-42=0$ or $x+3=0$

$\Rightarrow x=42$ or $x=-3$

Since, speed cannot be negative, we reject $x=-3$.

Hence, $x=42$

Thus, the original average speed of the train is 42 km/hr.

28. Let the four consecutive numbers in an A.P. be (a – 3d), (a – d), (a + d) and (a + 3d).

Sum of the numbers = 32

$\Rightarrow (a-3d)+(a-d)+(a+d)+(a+3d)=32$

$\Rightarrow 4a=32$

$\Rightarrow a=8$

It is given that,

$\frac{(a-3d)(a+3d)}{(a-d)(a+d)}=\frac{7}{15}$

$\Rightarrow \frac{a^2-9d^2}{a^2-d^2}=\frac{7}{15}$

$\Rightarrow \frac{64-9d^2}{64-d^2}=\frac{7}{15}$

$\Rightarrow 960-135d^2=448-7d^2$

$\Rightarrow 128d^2=512$

$\Rightarrow d^2=4$

$\Rightarrow d=\pm 2$

$\Rightarrow a-3d=8-3(2)=8-6=2$

$a-d=8-2=6$

$a+d=8+2=10$

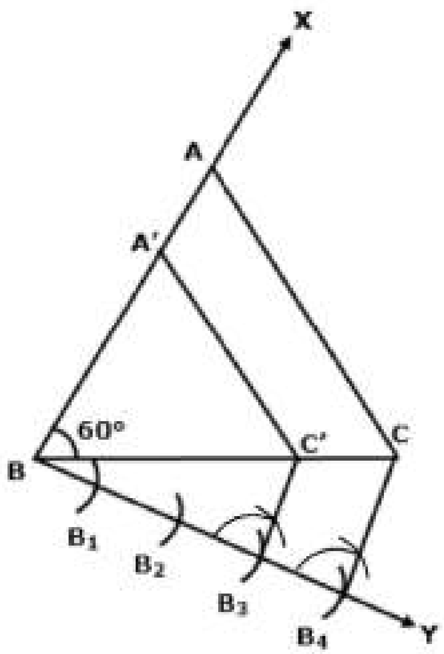
$a+3d=8+3(2)=8+6=14$

Thus, the four numbers are 2, 6, 10, 14.

29. Steps of construction:

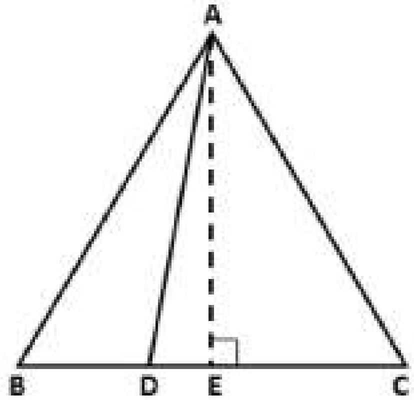
1) Draw a line segment BC = 6 cm
2) At B, construct $\angle XBC = 60^\circ$
3) With B as centre and radius 5 cm, draw an arc intersecting XB at A.
4) Join AC to obtain ΔABC.
5) Below BC, make an acute $\angle CBY$.
6) Along BY, mark off 4 points $\left(\text{greater of 3 and 4 in } \frac{3}{4}\right)$ B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$
7) Join B_4C
8) From point B_3, draw a line parallel to B_4C intersecting BC at C'
9) From point C', draw a line parallel to CA intersecting AB at A'

Thus, $\Delta A'BC'$ is the required triangle.



30. Let ABC be an equilateral triangle and let D be a point on BC such that $BD = \frac{1}{3}BC$.

Construction: Draw AE ⊥ BC. Join AD.



ΔABC is an equilateral triangle.

$\Rightarrow$ E is the mid-point of BC.

$\therefore BE = CE = \frac{1}{2}BC$

In right-angle ΔAEB, by Pythagoras theorem,

$AB^2 = AE^2 + BE^2$ ....(i)

In right-angle ΔAED, by Pythagoras theorem,

$AD^2 = AE^2 + DE^2$ ....(ii)

Subtracting (ii) from (i), we have

$AB^2 - AD^2 = (AE^2 + BE^2) - (AE^2 + DE^2)$

$\Rightarrow AB^2 - AD^2 = BE^2 - DE^2$

$\Rightarrow AB^2 - AD^2 = \left(\frac{1}{2}AB\right)^2 - \left(\frac{1}{6}AB\right)^2$ $\quad [DE = BE - BD = \frac{1}{2}AB - \frac{1}{3}AB = \frac{1}{6}AB]$

$\Rightarrow AB^2 - AD^2 = \frac{1}{4}AB^2 - \frac{1}{36}AB^2$

$\Rightarrow AB^2 - AD^2 = \frac{8}{36}AB^2$

$\Rightarrow AB^2 - AD^2 = \frac{2}{9}AB^2$

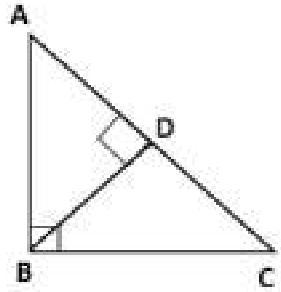
$\Rightarrow 9AB^2 - 9AD^2 = 2AB^2$

$\Rightarrow 7AB^2 = 9AD^2$

$\Rightarrow 9(AD)^2 = 7(AB)^2$

OR

Consider the following figure:



Given : In ΔABC, $\angle ABC = 90^\circ$

To prove: $AC^2 = AB^2 + BC^2$

Construction: Draw seg $BD \perp$ hypotenuse AC

and $A - D - C$

Proof:

In ΔABC, seg $BD \perp$ hypotenuse AC ....(construction)

$\therefore \Delta ABC \sim \Delta ADB$ ....(Similarity in right angled triangles)

$\therefore \frac{AB}{AD} = \frac{AC}{AB}$ ....(Corresponding sides of similar triangles)

$\therefore AB^2 = AC \times AD$ ....(i)

Similarly, $\Delta ABC \sim \Delta BDC$ ....(Similarity in right angled triangles)

$\therefore \frac{BC}{DC} = \frac{AC}{BC}$ ....(Corresponding sides of similar triangles)

$\therefore BC^2 = AC \times DC$ ....(ii)

$AB^2 + BC^2 = AC \times AD + AC \times DC$ ....[Adding equations (i) and (ii)]

$= AC(AD + DC)$

$= AC \times AC$

$= AC^2$

$\therefore AC^2 = AB^2 + BC^2$