**CBSE**
**Class X Mathematics**
**Board Paper – 2019**
**All India Set – 3**

**Time: 3 hours** **Total Marks: 80**

**General Instructions:**

(i) **All** questions are compulsory.

(ii) The question paper consists of **30** questions divided into four sections – A, B, C and D.

(iii) Section A contains **6** questions of **1** mark each. Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section D contains **8** questions of **4** marks each.

(iv) There is no overall choice. However, an internal choice has been provided in **two** questions of **1** marks each and, **two** questions of **2** marks each, **four** questions of **3** marks each and **three** questions of **4** marks each. You have attempt only **one** of the alternative in all such question.

(v) Use of calculators is **not** permitted.

## SECTION A

Question numbers 1 to 6 carry 1 mark each.

**1.** In Figure 1, PS = 3 cm. QS = 4 cm, $\angle PRQ = \theta$, $\angle PSQ = 90^\circ$, $PQ \perp RQ$ and RQ = 9 cm. Evaluate tan θ.

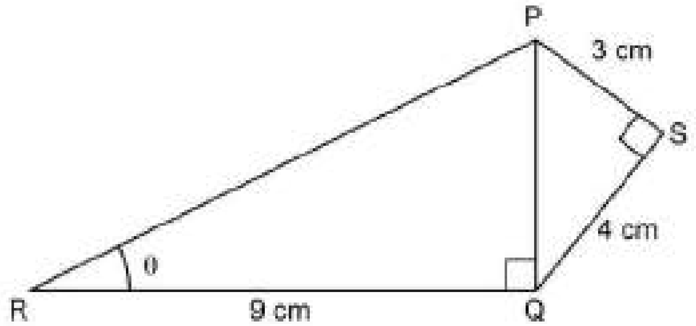


Figure 1

**OR**

If $\tan \alpha = \frac{5}{12}$, find the value of sec α.

**2.** Two concentric circles of radii a and b (a > b) are given. Find the length of the chord of the larger circle which touches the smaller circle.

**3.** Find the value(s) of x, if the distance between the points A(0,0) and B(x, -4) is 5 units.

**4.** Find after how many places of decimal the decimal form of the number $\frac{27}{2^3.5^4.3^2}$ will terminate.

**OR**

Express 429 as a product of its prime factors.

**5.** Write the discriminant of the quadratic equation $(x + 5)^2 = 2(5x - 3)$.

**6.** Find the sum of the first 10 multiples of 3.

## SECTION B

Question numbers 7 to 12 carry 2 marks each.

**7.** If HCF of 65 and 117 is expressible in the form 65n – 117, then find the value of n.

**OR**

On a morning walk, three persons step out together and their steps measure 30 cm, 36 cm and 40 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

**8.** A die is thrown once. Find the probability of getting (i) a composite number, (ii) a prime number.

**9.** Using completing the square method, show that the equation $x^2 - 8x + 18 = 0$ has no solution.

**10.** Cards numbered 7 to 40 were put in a box. Poonam selects a card at random. What is the probability that Poonam selects a card which is a multiple of 7?

**11.** Solve the following pair of linear equations:

$$3x + 4y = 10$$
$$2x - 2y = 2$$

**12.** Points A(3, 1), B(5, 1) C(a, b) and D(4, 3) are vertices of a parallelogram ABCD. Find the values of a and b.

**OR**

Point P and Q trisect the line segment joining the point A(-2, 0) and B(0, 8) such that P is near to A. Find the coordinates of points P and Q.

## SECTION C

Question numbers 13 to 22 carry 3 marks each.

**13.** A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

| Number of days: | 0 - 6 | 6 - 12 | 12 - 18 | 18 - 24 | 24 – 30 | 30 - 36 | 36 - 42 |
|---|---|---|---|---|---|---|---|
| Number of students: | 10 | 11 | 7 | 4 | 4 | 3 | 1 |

**14.** In Figure 2, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.

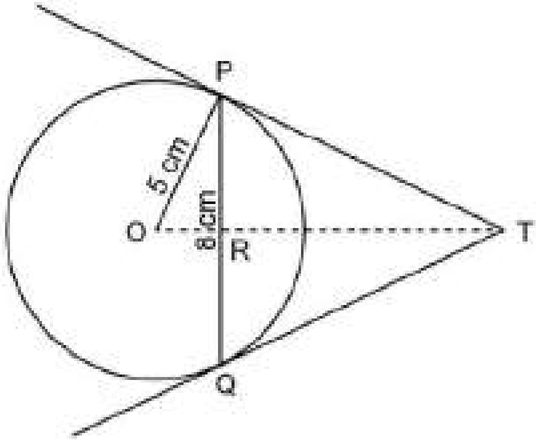


Figure 2

**OR**

Prove that opposite side of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**15.** A, B and C are interior angles of a triangle ABC. Show that

(i) $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

(ii) If $\angle A = 90^\circ$, then find the value of $\tan\left(\frac{B+C}{2}\right)$.

**OR**

If $\tan(A + B) = 1$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0^\circ < A + B < 90^\circ$, $A > B$, then find the values of A and B.

**16.** Prove that $\sqrt{3}$ is an irrational number.

**OR**

Find the largest number which on dividing 1251, 9377 and 15628 leaves remainders 1, 2 and 3 respectively.

**17.** Draw the graph of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Using this graph, find the values of x and y which satisfy both the equations.

**18.** Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes if 8 cm of standing water is needed?

**19.** The perpendicular from A on side BC of a $\Delta$ ABC meets BC at D such that DB = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$.

**OR**

AD and PM are medians of triangles ABC and PQR respectively where $\Delta ABC \sim \Delta PQR$. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

**20.** A chord of a circle of radius 14 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment of the circle.

$\left(\text{Use } \pi = \frac{22}{7} \text{ and } \sqrt{3} = 1.73\right)$

**21.** Find the value of k so that area of triangle ABC with A(k + 1, 1), B(4, -3) and C(7, -k) is 6 square units.

**22.** If $\frac{2}{3}$ and -3 are the zeroes of the polynomial $ax^2 + 7x + b$, then find the values of a and b.

**SECTION D**

Question numbers 23 to 30 carry 4 marks each.

**23.** Change the following distribution to a 'more than type' distribution.
Hence draw the 'more than type' ogive for this distribution.

| Class interval: | 20 - 30 | 30 - 40 | 40 - 50 | 50 – 60 | 60 - 70 | 70 - 80 | 80 - 90 |
|---|---|---|---|---|---|---|---|
| Frequency: | 10 | 8 | 12 | 24 | 6 | 25 | 15 |

**24.** The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower. (Given $\sqrt{3} = 1.732$)

**25.** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

**OR**

Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

**26.** If m times the $m^{th}$ term of an Arithmetic Progression is equal to n times its $n^{th}$ term and $m \neq n$, show that the $(m + n)^{th}$ term of the A.P. is zero.

**OR**

The sum of the first three numbers in an Arithmetic Progression is 18. If the product of the first and the third term is 5 times the common difference, find the three numbers.

**27.** In Figure 3, a decorative block is shown which is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge 6 cm and the hemisphere fixed on the top has a diameter of 4.2 cm. Find

(a) the total surface area of the block.

(b) the volume of the block formed. (Take $\pi = \frac{22}{7}$)

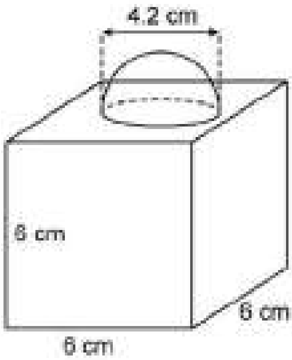


Figure 3

**OR**

A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 $cm^3$. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (Use $\pi$ = 3.14)

**28.** Construct a triangle, the lengths of whose sides are 5 cm, 6 cm and 7 cm. Now construct another triangle whose sides are $\frac{5}{7}$ times the corresponding sides of the first triangle.

**29.** Prove that :

$$\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\,\text{cosec}\,\theta - 2\sin\theta\cos\theta.$$

**30.** A motorboat whose speed in still water is 9 km/h, goes 15 km downstream and comes back to the same spot, in a total time of 3 hours 45 minutes. Find the speed of the stream.

**CBSE**

**Class X Mathematics**

**Board Paper – 2019 Solution**

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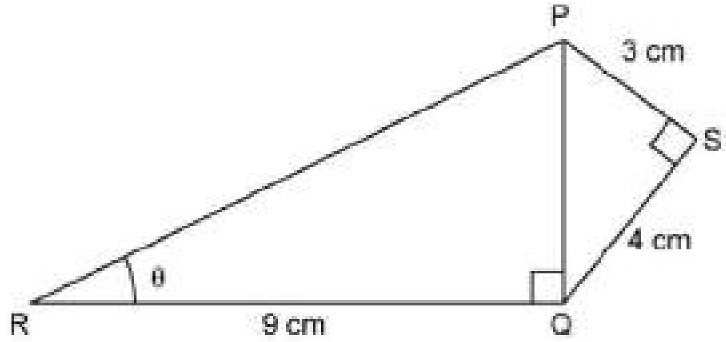
## Section A

**1.**




In the figure we have a right angled triangle $\Delta PSQ$

So on applying Pythagoras theorem we get

$\therefore PQ^2 = PS^2 + QS^2$

$$\therefore PQ = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

In $\Delta PQR$

$$\therefore \tan\theta = \frac{PQ}{QR}$$

$$\therefore \tan\theta = \frac{5}{9}$$

**OR**

Given :

$\tan\alpha = \frac{5}{12}$

we know,

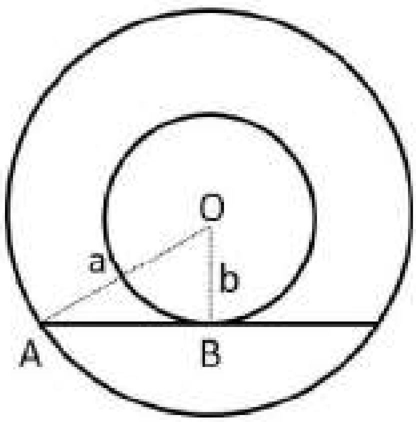
$\sec^2\alpha = 1 + \tan^2\alpha$

$\sec\alpha = \sqrt{1 + \left(\frac{5}{12}\right)^2}$

$\sec\alpha = \sqrt{\frac{144+25}{144}} = \sqrt{\frac{169}{144}} = \frac{13}{12}$

$\sec\alpha = \frac{13}{12}$

**2.**




It is given that a > b,

AB is the tangent to the smaller circle,

OB ⊥ AB.....(tangent ⊥ radius)

So ΔABO is right angled triangle

$\therefore AO^2 = AB^2 + BO^2$

$\therefore AB = \sqrt{a^2 - b^2}$

Now we know that the perpendicular drawn from the center to a chord bisects the chord so

The length of chord will be $2AB = 2\sqrt{a^2 - b^2}$

**3.**

A(0, 0) and B(x, -4)

The distance AB is given by distance formula

$$AB = \sqrt{(x-0)^2 + (-4-0)^2}$$
$$AB = \sqrt{x^2 + 16}$$

Now,

AB = 5 units

$$\therefore 5 = \sqrt{x^2 + 16}$$

squaring,

$$\therefore 25 = x^2 + 16$$
$$\therefore x^2 = 9$$
$$\therefore x = \pm 3$$

**4.**

$$\frac{27}{2^3.5^4.3^2}$$
$$= \frac{3^3}{2^3.5^4.3^2}$$
$$= \frac{3}{2^3.5^4}$$

Now power of 5 is 4 and that of 2 is 3

$\therefore 4 > 3$, so the decimal will terminate after 4 places

**OR**

| | |
|---|---|
| 13 | 429 |
| 11 | 33 |
| 3 | 3 |
| | 1 |

$$\therefore 429 = 3 \times 11 \times 13$$

**5.**

$$(x+5)^2 = 2(5x-3)$$
$$\therefore x^2 + 10x + 25 = 10x - 6$$
$$\therefore x^2 + 31 = 0$$
$$\text{discriminant} = b^2 - 4ac$$
$$\therefore b^2 - 4ac = 0^2 - 4(1)(31) = -124$$

**6.**

The multiples of 3 are

3, 6, 9, 12...

The above series is in arithmetic progression

$\therefore a = 3$ and $d = 3$

We need sum of 10 multiples

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$\therefore S_{10} = \frac{10}{2}(2 \times 3 + (10-1)3)$$
$$\therefore S_{10} = 165$$

## Section B

**7.**

$$65 = 5 \times 13$$
$$117 = 3 \times 3 \times 13$$
$$\text{HCF} = 13$$
Given that
$$\text{HCF} = 65n - 117$$
$$\therefore 13 = 65n - 117$$
$$\therefore 130 = 65n$$
$$\therefore n = 2$$

**OR**

We have to find the LCM of 30 cm, 36 cm and 40 cm to get the required minimum distance. Because we are asked the minimum distance

Now,

$30 = 3 \times 2 \times 5,$

$36 = 2 \times 3 \times 2 \times 3$

$40 = 2 \times 2 \times 2 \times 5$

$\therefore$ LCM $(30, 36, 40) = 2^3 \times 3^2 \times 5 = 360$

Minimum distance each should walk 360 cm. so that, each can cover the same distance in complete steps.

**8.**

A die is thrown,

So sample space S is given by

$\therefore S = \{1, 2, 3, 4, 5, 6\}$

$\therefore n(S) = 6$

Let A be the event such that composite number shows up

$\therefore A = \{4, 6\}$

$\therefore n(A) = 2$

(i) Probability of getting composite number

$\therefore P(A) = \frac{2}{6} = \frac{1}{3}$

Let B be the event such that prime number shows up

$\therefore B = \{2, 3, 5\}$

$\therefore n(B) = 3$

(ii) Probability of getting prime number

$\therefore P(B) = \frac{3}{6} = \frac{1}{2}$

**9.**

The given quadratic equation is

$x^2 - 8x + 18 = 0$

The above equation can be written as

$\therefore x^2 - 8x + 16 - 16 + 18 = 0$

$\therefore x^2 - 8x + 16 = -2$

$\therefore (x - 4)^2 = -2$

$\therefore x - 4 = \sqrt{-2}$

Which is not possible, so the given quadratic equation has no roots.

**10.**

Poonam selects a card in random from 7 to 40

So the sample space S is

$\therefore S = \{7, 8, 9, 10, \ldots\ldots 40\}$

$\therefore n(S) = 34$

Let A be the event of getting multiple of 7

$\therefore A = \{7, 14, 21, 28, 35\}$

$\therefore n(A) = 5$

Probability of getting multiple of 7

$\therefore P(A) = \dfrac{5}{34}$

**11.**

$3x + 4y = 10$ .... (i)

$2x - 2y = 2$ .... (ii)

Multiply equation (ii) by 2, we get

$4x - 4y = 4$ .... (iii)

Adding equation (i) and (iii), we get

$\therefore 7x = 14$

$\Rightarrow x = 2$

Put x = 2 in the equation (ii), we get

$\therefore 2(2) - 2y = 2$

$\Rightarrow 2y = 4 - 2$

$\Rightarrow \quad y = \frac{2}{2} = 1$

Hence the solutions is x = 2 and y = 1.

**12.**

Points A (3, 1), B(5, 1), C(a, b) and D(4, 3) are vertices of parallelogram

We know that diagonals of parallelogram bisect each other

The M be the midpoint of both AC and BD

So by midpoint formula

$\therefore M = \left(\frac{3+a}{2}, \frac{1+b}{2}\right)$

also,

$M = \left(\frac{5+4}{2}, \frac{1+3}{2}\right) = \left(\frac{9}{2}, 2\right)$

so,

$\therefore \left(\frac{3+a}{2}, \frac{1+b}{2}\right) = \left(\frac{9}{2}, 2\right)$

$\therefore \frac{3+a}{2} = \frac{9}{2}$

$\therefore a = 6$

$\therefore \frac{1+b}{2} = 2$

$b = 3$

**OR**

P and Q trisect AB, given that A (-2, 0) and B (0, 8)


Now P will divide AB in ratio 1: 2

So by using section formula we get

$$P(x,y)=\left(\frac{2\times -2+1\times 0}{1+2},\frac{2\times 0+1\times 8}{1+2}\right)$$

$$\therefore P(x,y)=\left(\frac{-4}{3},\frac{8}{3}\right)$$

Now P is the midpoint of AQ

Let coordinates of Q be (a, b)

So by using midpoint theorem

$$P\left(\frac{-4}{3},\frac{8}{3}\right)=\left(\frac{-2+a}{2},\frac{0+b}{2}\right)$$

$$\frac{-4}{3}=\frac{-2+a}{2}$$

$$\therefore a=\frac{-2}{3}$$

$$\frac{8}{3}=\frac{b}{2}$$

$$\therefore b=\frac{16}{3}$$

$\therefore$ The coordinates of Q are $\left(\frac{-2}{3},\frac{16}{3}\right)$

## Section C

**13.**

We will use direct method to find the mean:

| No. of days | No. of students | Class marks($x_i$) | $f_i x_i$ |
|---|---|---|---|
| 0-6 | 10 | $\frac{0+6}{2}=3$ | 30 |
| 6-12 | 11 | $\frac{6+12}{2}=9$ | 99 |
| 12-18 | 7 | $\frac{12+18}{2}=15$ | 105 |
| 18-24 | 4 | $\frac{18+24}{2}=21$ | 84 |
| 24-30 | 4 | $\frac{24+30}{2}=27$ | 108 |
| 30-36 | 3 | $\frac{30+36}{2}=33$ | 99 |
| 36-42 | 1 | $\frac{36+42}{2}=39$ | 39 |
| | $\sum f_i = 40$ | | $\sum f_i x_i = 564$ |

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{564}{40} = 14.1$$

Hence, the number of days a student was absent is 14.1

**14.**

Let TP = x.

In $\triangle PRT$ and $\triangle QRT$

PT = TQ ...(tangents drawn from external point are equal in length)

RT ....common side

$\angle PTR = \angle QTR$...(the line joining the center of circle and the external point bisects the angle between the tangents, which are drawn from that external point)

$\triangle PRT \cong \triangle QRT$...( SAS test )

$\angle PRT = \angle QRT$...( C.P.C.T.)

Also,

$\angle PRT = \angle QRT = 90°$...(linear pair)

We know that the perpendicular drawn from the centre of the circle to a chord bisects it.

Therefore, PR = RQ = 4 cm and OR $\perp$ PQ
Using Pythagoras theorem in right angled triangle PRO, we have

$$(OP)^2 = (PR)^2 + (OR)^2$$
$$\Rightarrow 5^2 = 4^2 + (OR)^2$$
$$\Rightarrow (OR)^2 = 5^2 - 4^2$$
$$\Rightarrow (OR)^2 = 25 - 16 = 9$$
$$\Rightarrow OR = 3 \text{ cm}$$

Now, again using Pythagoras theorem in triangle PRT we have

$$(PT)^2 = (PR)^2 + (RT)^2$$
$$\Rightarrow x^2 = 4^2 + RT^2 \ldots.(i)$$

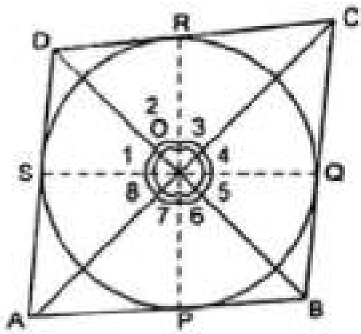
Since we know that the radius drawn to tangent is perpendicular to the tangent so, we have
In right triangle OPT,

$$(OT)^2 = (OP)^2 + (PT)^2$$
$$\Rightarrow (OT)^2 = 5^2 + x^2$$
$$\Rightarrow (OR + RT)^2 = 5^2 + x^2$$
$$\Rightarrow (3 + RT)^2 = 5^2 + 4^2 + RT^2 \ldots \text{Using (i)}$$
$$\Rightarrow 9 + RT^2 + 6RT = 25 + 16 + RT^2$$
$$\Rightarrow 9 + 6RT = 41$$
$$\Rightarrow 6RT = 32$$
$$\Rightarrow RT = \frac{16}{3}$$

Again using (i), we have

$$TP^2 = x^2 = 16 + RT^2$$
$$\Rightarrow TP^2 = 16 + \left(\frac{16}{3}\right)^2$$
$$\Rightarrow TP^2 = 16 + \frac{256}{9}$$
$$\Rightarrow TP^2 = \frac{144 + 256}{9}$$
$$\Rightarrow TP^2 = \frac{400}{9}$$
$$\Rightarrow TP = \frac{20}{3} \text{cm}$$

**OR**




Let ABCD be a quadrilateral circumscribing a circle with O such that it touches the circle at point P, Q, R, S.

Join the vertices of the quadrilateral ABCD to the center of the circle.

In ΔOAP and ΔOAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the circle)

OA = OA (Common side)

ΔOAP ≅ ΔOAS (SSS congruence condition)

∴ ∠POA = ∠AOS

⇒∠7 = ∠8

Similarly we get,

∠4 = ∠3

∠5 = ∠6

∠1 = ∠2

Adding all these angles,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360^\circ$

$\Rightarrow 2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360^\circ$

$\Rightarrow 2(\angle 1 + \angle 8) + 2(\angle 4 + \angle 5) = 360^\circ$

$\Rightarrow (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$

$\Rightarrow \angle AOD + \angle COB = 180^\circ$

Similarly, we can prove that $\angle AOB + \angle COD = 180^\circ$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**15.**

In triangle ABC, sum of all interior angles is $180^\circ$

$\Rightarrow A + B + C = 180^\circ$

$\Rightarrow B + C = 180^\circ - A$ ... (a)

(i)

$\Rightarrow \frac{B+C}{2} = \frac{180^\circ - A}{2}$

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{180^\circ - A}{2}\right)$

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$

Since, $\sin(90^\circ - \theta) = \cos\theta$

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$

(ii)

Using equation (a), we have

$B + C = 180^\circ - A$

Since, $\angle A = 90^\circ$

$\Rightarrow B + C = 180^\circ - 90^\circ$

$\Rightarrow B + C = 90^\circ$

$\Rightarrow \frac{B+C}{2} = 45^\circ$

$\Rightarrow \tan\left(\frac{B+C}{2}\right) = \tan 45^\circ$

$\Rightarrow \tan\left(\frac{B+C}{2}\right) = 1$

**OR**

Given: tan (A + B) = 1 ..... (i)

And $\tan(A - B) = \frac{1}{\sqrt{3}}$ ...... (ii)

Using first equation, we have

$\tan(A + B) = 1 = \tan 45^\circ$

$\Rightarrow A + B = 45^\circ$ ..... (iii)

Since A > B, so $A - B > 0$

Now, using (ii)

$\tan(A - B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$\Rightarrow A - B = 30^\circ$ ..... (iv)

Solving (iii) and (iv), we have

$2A = 75^\circ \Rightarrow A = 37.5^\circ$

Thus, we have $B = 7.5^\circ$

**16.**

Prove that $\sqrt{3}$ is an irrational number.

Let us assume that $\sqrt{3}$ is a rational number.

That is, we can find integers a and b ($b \neq 0$) such that $\sqrt{3} = \frac{a}{b}$

Suppose a and b have a common factor other than 1, then we can divide by the common factor, and assume that a and b are coprime.

$\sqrt{3}b = a \Rightarrow 3b^2 = a^2$ (Squaring on both the sides) .... (i)

Therefore, $a^2$ is divisible by 3

Hence 'a' is also divisible by 3.

So, we can write a = 3c for some integer c.

Equation (1) becomes,

$3b^2 = (3c)^2$

$\Rightarrow 3b^2 = 9c^2$

$\therefore b^2 = 3c^2$

This means that $b^2$ is divisible by 3, and so b is also divisible by 3.

Therefore, a and b have at least 3 as a common factor.

But this contradicts the fact that a and b are coprime.

This contradiction has arisen because of our incorrect assumption that √3 is rational.

So, we conclude that √3 is irrational.

**OR**

Let the largest number which divides the given numbers be x.

Since, it leaves the remainders 1, 2 and 3 while dividing 1251, 9377 and 15628

So, the numbers which are divisible by x are:

1251 - 1 =1250, 9377 - 2 =9375 and 15628 - 3 =15625

Here, x is the largest number dividing these 3 numbers so x becomes the GCD or HCF

Now, $1250 = 5^4 \times 2$

$9375 = 5^5 \times 3$ and $15625 = 5^6$

Therefore x = HCF (1250, 9375, 15625) = $5^4$ = 625.

Hence, the required number is 625.

**17.**

The given equations are:

$x - y + 1 = 0$ ...... (i)

$3x + 2y - 12 = 0$...... (ii)

To draw the graph of these equations, let us find some points lying on them:

For equation (i)

| | | | | |
|---|---|---|---|---|
| x | 0 | 1 | -1 | 2 |
| y | 1 | 2 | 0 | 3 |

Plot these points i.e. (0, 1), (1, 2), (-1, 0) and (2, 3) on the graph

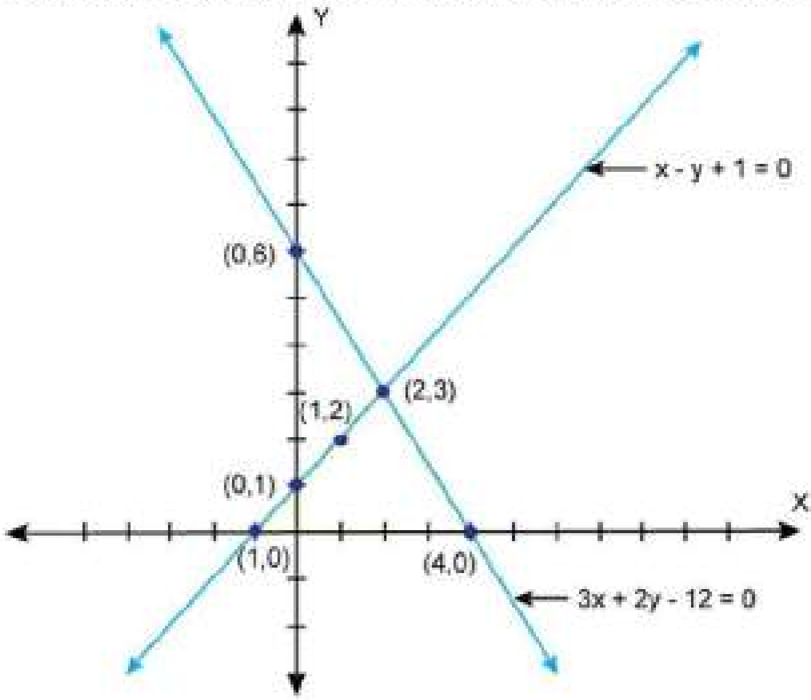
Now, draw a straight line connecting them and extend the line.

For equation (ii)

| | | | | |
|---|---|---|---|---|
| x | 0 | 1 | 4 | 2 |
| y | 6 | $\frac{9}{2}$ | 0 | 3 |

Plot these points i.e. (0, 6), $\left(1, \frac{9}{2}\right)$, (4, 0) and (2, 3) on the graph

Now, draw a straight line connecting them and extend the line.




Clearly, the two lines intersect at a point (2, 3).

Thus, x = 2 and y = 3 satisfy both the equations.

**18.**

Speed of flowing of water from canal is 10 km/h
This means the length of water flows in 1 hour = 10 km
$\Rightarrow$ length of water flows in 30 minutes = 5 km

Now,
volume of water flowing from canal = length of water flows in 30 minutes × breadth of canal × depth of canal
= 5000 m × 6 m × 1.5 m = 45000 $m^3$

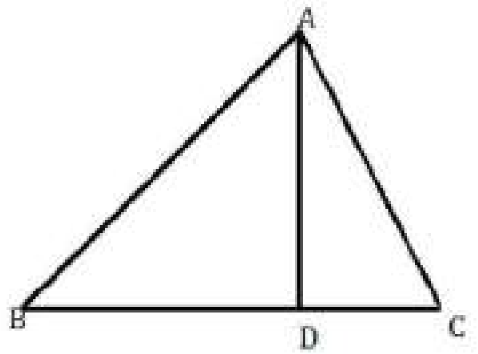
Let the area of field = x $m^2$
Then, volume of water irrigates into the field = area of field × height of water during irrigation
= x $m^2 \times 8\times 10^{-2}$ m = 0.08x $m^3$

Now, volume of water flowing from canal = volume of water in field
45000 $m^3$ = 0.08x $m^3$
x = 562500 $m^2$

We know, 1 hectare = 10000 $m^2$
So, area of field in hectare = 562500/10000 = 56.25 hectares.

**19.**

Given that in $\Delta ABC$, we have




$AD \perp BC$ and $BD = 3CD$

Now, using Pythagoras theorem in right angle triangles ADB and ADC, we have

$AB^2 = AD^2 + BD^2$ ....... (i)

$AC^2 = AD^2 + DC^2$ ....... (ii)

Subtracting equation (ii) from equation (i), we get

$AB^2 - AC^2 = BD^2 - DC^2$

$= (3CD)^2 - DC^2$

$= 9DC^2 - DC^2$

$= 8DC^2$

Since, $BC = BD + DC$

$\Rightarrow BC = 3DC + DC$

$\Rightarrow BC = 4DC$

$\Rightarrow DC = \frac{BC}{4}$

Then, we have

$AB^2 - AC^2 = 8\left(\frac{BC}{4}\right)^2$

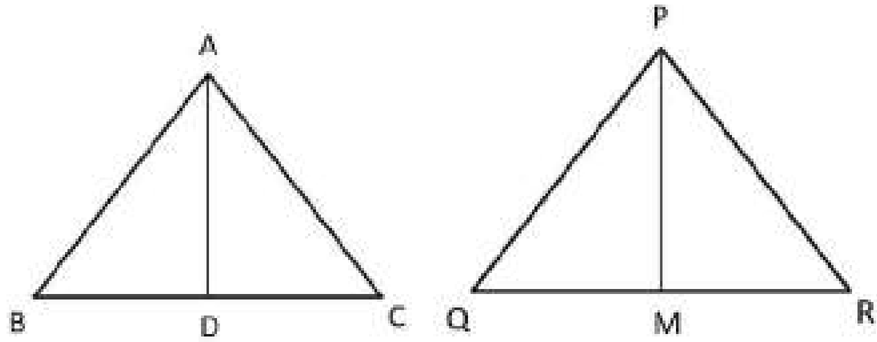
$\Rightarrow AB^2 - AC^2 = \frac{BC^2}{2}$

$\Rightarrow 2AB^2 - 2AC^2 = BC^2$

$\Rightarrow 2AB^2 = 2AC^2 + BC^2$

Hence, proved.

**OR**




Given: AD and PM are medians of triangle ABC and PQR respectively.

$\Rightarrow BD = DC = \frac{1}{2}BC$...... (i)

$QM = MR = \frac{1}{2}QR$ ...... (ii)

Since $\triangle ABC \sim \triangle PQR$, using similarity criterion

$\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$

Also, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{2BD}{2QM} = \frac{BD}{QM}$ ...... Using (i) and (ii)

$\Rightarrow \frac{PQ}{AB} = \frac{QM}{BD}$

Therefore in triangles ABD and PQM,

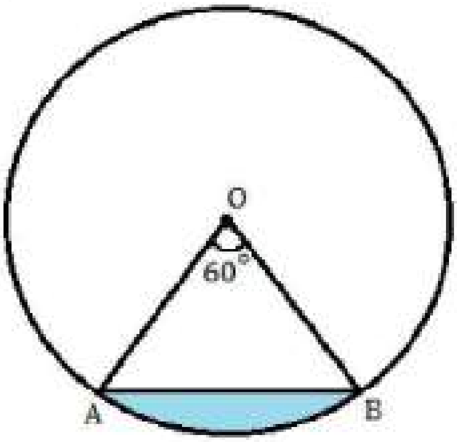
$\frac{AB}{PQ} = \frac{BD}{QM}$ and $\angle B = \angle Q$

$\therefore \triangle ABD \sim \triangle PQM$ ....... By SAS

$\therefore \frac{AB}{PQ} = \frac{AD}{PM}$ ...... (Since, sides of similar triangles are in proportion)

Hence, proved.

**20.**




Radius of the circle = OA = OB = 14 cm

Let $\theta$ be the angle at the centre = 60°

Now in $\Delta AOB$,

$\angle A = \angle B$........ (Angles opposite to equal sides are equal)

By the angle sum property of triangle,

$\angle A + \angle B + \angle O = 180°$

$x + x + 60° = 180°$

$\Rightarrow 2x = 120°$

$\Rightarrow x = 60°$

Therefore, all the angles of a triangle AOB is 60°

So, $\Delta AOB$ is equilateral

$\Rightarrow OA = OB = AB$

Area of minor segment = Area of sector – Area of triangle AOB

$= \frac{\theta}{360°} \times \pi r^2 - \frac{\sqrt{3}}{4}(AB)^2$

$= \frac{60°}{360°} \times \pi(14)^2 - \frac{\sqrt{3}}{4}(14)^2$

$= \frac{1}{6} \times \frac{22}{7} \times 14 \times 14 - \frac{\sqrt{3}}{4} \times 14 \times 14$

$= \frac{308}{3} - 49\sqrt{3}$

$= \frac{308 - 147\sqrt{3}}{3}$ cm$^3$

**21.** Given: A(k + 1, 1), B(4, -3) and C(7, - k)

Area of triangle ABD is 6 square units

We know that the area of triangle is given by

$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Rightarrow 6 = \frac{1}{2}[(k+1)(-3+k) + 4(-k-1) + 7(1+3)]$$

$$\Rightarrow 12 = (k+1)(-3+k) + 4(-k-1) + 7(1+3)$$

$$\Rightarrow 12 = -3k + k^2 - 3 + k - 4k - 4 + 28$$

$$\Rightarrow 12 = k^2 - 6k + 21$$

$$\Rightarrow k^2 - 6k + 9 = 0$$

$$\Rightarrow (k-3)^2 = 0$$

$$\Rightarrow k = 3$$

**22.** Given polynomial is $p(x) = ax^2 + 7x + b$

$\frac{2}{3}$ and -3 are the zeroes of p(x)

$$\Rightarrow p\left(\frac{2}{3}\right) = 0 \text{ and } p(-3) = 0$$

$$\Rightarrow a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$\Rightarrow \frac{4a}{9} + \frac{14}{3} + b = 0$$

$$\Rightarrow 4a + 9b + 42 = 0 \text{...... (i)}$$

Also, $a(-3)^2 + 7(-3) + b = 0$

$$\Rightarrow 9a + b - 21 = 0 \text{ ...... (ii)}$$

Solving (i) and (ii) simultaneously, we get

$a = 3$

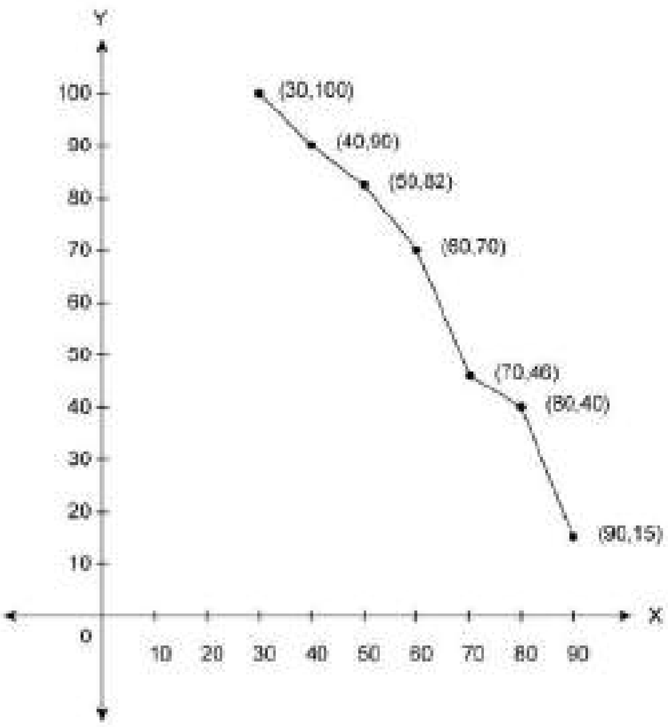
Substituting the value of a in (ii), we get

$b = -6$

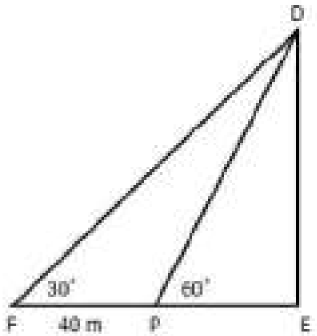
## Section D

23.

| Intervals | f | cf(more than type) | |
|---|---|---|---|
| 20-30 | 10 | 100 | (30, 100) |
| 30-40 | 8 | 90 | (40, 90) |
| 40-50 | 12 | 82 | (50, 82) |
| 50-60 | 24 | 70 | (60, 70) |
| 60-70 | 6 | 46 | (70, 46) |
| 70-80 | 25 | 40 | (80, 40) |
| 80-90 | 15 | 15 | (90, 15) |
| | 100 | | |



24.




From the figure, DE is the tower.

When sun's altitude is 60° then ∠ DPE = 60° and length of shadow is PE.

Shadow is 40 m more when angle changes from 60° to 30°

FP = 40 m.

As tower is a vertical ground, ∠ DEF is 90°.

In right angled triangle DEF,

$\tan P = \frac{DE}{PE}$

$\therefore \tan 60° = \frac{DE}{PE}$

$\therefore \sqrt{3} = \frac{DE}{PE}$

$\therefore PE = \frac{DE}{\sqrt{3}}$ .....(i)

In right angled triangle DEF,

$\tan F = \frac{DE}{FE}$

$\therefore \tan 30° = \frac{DE}{FE}$

$\therefore \frac{1}{\sqrt{3}} = \frac{DE}{FE}$

$\therefore FE = \sqrt{3}DE$

$\therefore FP + PE = \sqrt{3}DE$

$\therefore 40 + PE = \sqrt{3}DE$

$\therefore PE = \sqrt{3}DE - 40$ ....(ii)

$\frac{DE}{\sqrt{3}} = \sqrt{3}DE - 40$ from (i) and (ii)

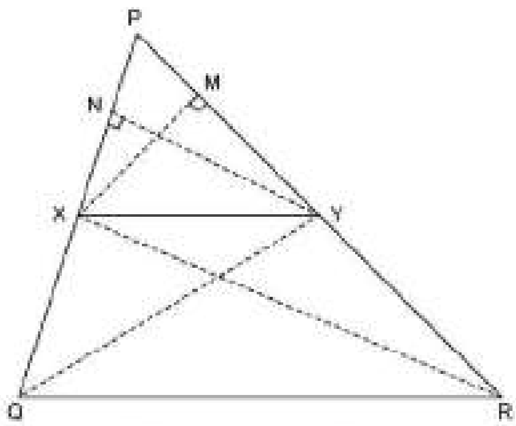
$\therefore DE = 3DE - 40\sqrt{3}$

$\therefore 2DE = 40\sqrt{3}$

$\therefore DE = 20\sqrt{3}$

Hence, the height of the tower = DE = $20\sqrt{3}$ m.

25.




Given : ΔPQR in which XY || QR, XY intersects PQ and PR at X and Y respectively.

To prove : $\frac{PX}{XQ} = \frac{PY}{YR}$

Construction : Join RX and QY and draw YN perpendicular to PQ and XM perpendicular to PR.

Proof :

Since, ar(ΔPXY) = $\frac{1}{2} \times PX \times YN$ .....(i)

ar(ΔPXY) = $\frac{1}{2} \times PY \times XM$ ....(ii)

Similarly, ar(ΔQXY) = $\frac{1}{2} \times QX \times NY$ .....(iii)

ar(ΔRXY) = $\frac{1}{2} \times YR \times XM$ .....(iv)

Dividing (i) by (iii) we get,

$$\therefore \frac{ar(PXY)}{ar(QXY)} = \frac{\frac{1}{2} \times PX \times YN}{\frac{1}{2} \times QX \times YN} = \frac{PX}{QX} \quad ....(v)$$

Again dividing (ii) by (iv)

$$\therefore \frac{ar(PXY)}{ar(RXY)} = \frac{\frac{1}{2} \times PY \times XM}{\frac{1}{2} \times YR \times XM} = \frac{PY}{YR} \quad ....(vi)$$

Since the area of triangles with same base and between same parallel lines are equal, so

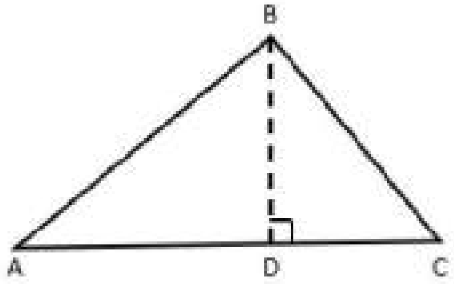
∴ ar(ΔQXY) = ar(ΔRXY) .....(vii)

As ΔQXY and ΔRXY are on same base XY and between same parallel lines XY and QR.

Therefore, from (v), (vi) and (vii) we get

$$\therefore \frac{PX}{XQ} = \frac{PY}{YR}$$

**OR**




Given : A right angle triangle ABC at B.

To prove : $AC^2 = AB^2 + BC^2$

Construction : Draw BD perpendicular to AC.

Proof :

If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

$\therefore \Delta ADB \sim \Delta ABC$

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$ $\because$ c. p. c. t.

$\therefore AD \times AC = AB^2$ .....(i)

$\Delta BDC \sim \Delta ABC$

$\therefore \frac{CD}{BC} = \frac{BC}{AC}$ $\because$ c. p. c. t.

$\therefore CD \times AC = BC^2$ .....(ii)

Adding (i) and (ii)

$\therefore AD \times AC + CD \times AC = AB^2 + BC^2$

$\therefore AC(AD + CD) = AB^2 + BC^2$

$\therefore AC \times AC = AB^2 + BC^2$

$\therefore AB^2 + BC^2 = AC^2$

**26.** Let the $m^{th}$ term of an Arithmetic Progression be $t_m$ and $n^{th}$ term be $t_n$.
Also, a and d be the first term and common difference respectively.
According to the question,
$mt_m = nt_n$
$\therefore m[a + (m - 1)d] = n[a + (n - 1)d]$
$\therefore am + m^2d - md = an + n^2d - nd = 0$
$\therefore a(m - n) + (m^2 - n^2)d - (m - n)d = 0$
$\therefore a(m - n) + (m + n)(m - n)d - (m - n)d = 0$
$\therefore (m - n)[a + (m + n)d - d] = 0$
It is given that $m \neq n$.
$\therefore a + (m + n - 1)d = 0$
$\therefore t_{m+n} = 0$

**OR**

Let the three numbers of an arithmetic progression be a – d, a, a + d.
According to the question,
$a - d + a + a + d = 18$
$\therefore 3a = 18$
$\therefore a = 6$
Hence, three numbers are 6 – d, 6, 6 + d
According to the question,
$\therefore (6 - d)(6 + d) = 5d$
$\therefore 36 - d^2 = 5d$
$\therefore d^2 + 5d - 36 = 0$
$\therefore (d + 9)(d - 4) = 0$
$\therefore d = -9$ and $d = 4$
If d = -9 then the three numbers are 6 + 9, 6, 6 – 9 i. e. 15, 6, -3.
If d = 4 then the three numbers are 6 – 4, 6, 6 + 4 i. e. 2, 6, 10.

**27.** Let the side of the cube be a.
$\therefore a = 6$ cm
Let radius of the hemisphere be r.
$\therefore r = 2.1$ cm
(a)
The total surface area of the block
= Total surface area of a cube – base area of hemisphere + Curved surface area of hemisphere
$= 6a^2 - \pi r^2 + 2\pi r^2$
$= 6 \times 6^2 + \pi r^2$
$= 216 + \frac{22}{7} \times 2.1^2$
$= 216 + 13.86$
$= 229.86$ cm$^2$

(b)
Volume of the clock
= Volume of a cube + volume of hemisphere

$= a^3 + \frac{2}{3}\pi r^3$

$= 6^3 + \frac{2}{3} \times \frac{22}{7} \times (2.1)^3$

= 216+19.404
= 235.404 $cm^3$

**OR**

Volume of the bucket = 12308.8 $cm^3$, $r_1$ = 20 cm and $r_2$ = 12 cm, h = ?

$\therefore$ Volume of bucket $= \frac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$\therefore \frac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right) = 12308.8$

$\therefore \frac{22 \times h}{7 \times 3}\left(20^2 + 12^2 + 20 \times 12\right) = 12308.8$

$\therefore \frac{22h}{21}(400 + 144 + 240) = 12308.8$

$\therefore \frac{22h}{21} \times 784 = 12308.8$

$\therefore h = \frac{12308.8 \times 21}{22 \times 784}$

$\therefore$ h = 15 cm

Surface area of the metal sheet used $= \pi r_2^2 + \pi(r_1 + r_2)l$

$l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{15^2 + 8^2} = 17$ cm

$\therefore$ Surface area of the metal sheet used $= \pi r_2^2 + \pi(r_1 + r_2)l$

$= 3.14 \times 12^2 + 3.14 \times 32 \times 17$

$= 2160.32$ $cm^2$

Hence, height of the bucket = 15 cm, area of the metal sheet used = 2160.32 $cm^2$.

**28.** Steps of construction :

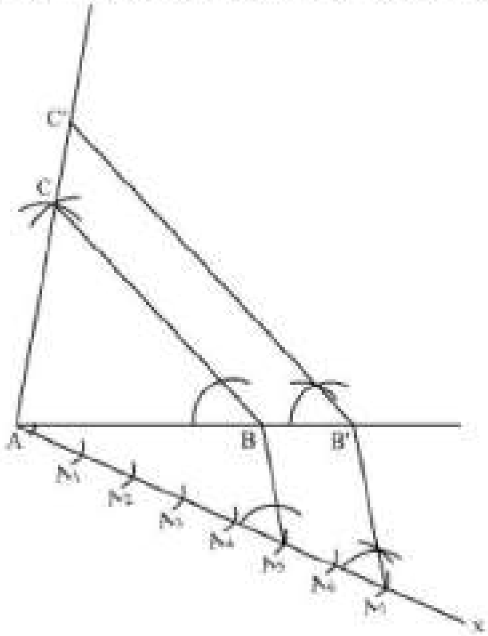
i. Draw a line segment AB of 5 cm. Taking A and B as centres, draw two arcs of 6 cm and 7 cm radius respectively. Let these arcs intersect each other at point C. ΔABC is the required triangle having length of sides as 5 cm, 6 cm and 7 cm respectively.

ii. Draw a ray AX making acute angle with the line AB on opposite side of vertex C.

iii. Locate 7 points $A_1$, $A_2$, $A_3$, $A_4$, $A_5$, $A_6$, $A_7$ (as 7 is greater between 5 and 7) on line AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

iv. Join $BA_5$ and draw a line through $A_7$ parallel to $BA_5$ to intersect extended line segment AB at point B'.

v. Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. ΔAB'C' is the required triangle.




**29.** LHS $= \dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta}$

$$= \frac{\frac{\sin^3\theta}{\cos^3\theta}}{\sec^2\theta} + \frac{\frac{\cos^3\theta}{\sin^3\theta}}{\operatorname{cosec}^2\theta}$$

$$= \frac{\frac{\sin^3\theta}{\cos^3\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^3\theta}{\sin^3\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \frac{\sin^3\theta}{\cos^3\theta} \times \cos^2\theta + \frac{\cos^3\theta}{\sin^3\theta} \times \sin^2\theta$$

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$$

$$= \frac{\sin^4\theta + \cos^4\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} - \frac{2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \sec\theta\ \text{cosec}\ \theta - 2\sin\theta\cos\theta$$

Hence, $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\ \text{cosec}\ \theta - 2\sin\theta\cos\theta$

**30.** Let T be the time for downstream and t be the time for upstream, travel.

T + t = 3 hrs 45 min = 15/4 hrs ....(i)

For downstream, distance/velocity = time

$\therefore \frac{15}{V+v} = T$ where V is the speed of boat and v is the speed of stream.

$\therefore \frac{15}{9+v} = T$ .....(ii)

Similarly,

$\therefore \frac{15}{9-v} = t$ ....(iii)

Adding (ii) and (iii)

$\therefore \frac{15}{9+v} + \frac{15}{9-v} = T + t$

$\therefore 15\left(\frac{9-v+9+v}{(9+v)(9-v)}\right) = \frac{15}{4}$ from (i)

$\therefore \frac{15\times18}{81-v^2} = \frac{15}{4}$

$\therefore \frac{18}{81-v^2} = \frac{1}{4}$

$\therefore 81 - v^2 = 72$

$\therefore v^2 = 9$

$\therefore v = 3$ km/hr

Hence, the speed of the stream is 3 km/hr.