Class IX Session 2024-25
Subject - Mathematics
Sample Question Paper - 1

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. The point which lies on x-axis at a distance of 3 units in the positive direction of x-axis is **[1]**

a) (0, -3) b) (0, 3)

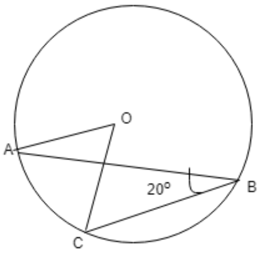
c) (3, 0) d) (-3, 0)

2. The length of the sides of a triangle are 5 cm, 7 cm and 8 cm. Area of the triangle is : **[1]**

a) $100\sqrt{3}$ cm^2 b) $10\sqrt{3}$ cm^2

c) 300 cm^2 d) $50\sqrt{3}$ cm^2

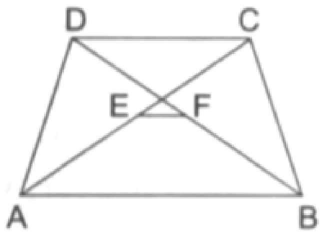
3. In the figure, O is the centre of the circle. If $\angle ABC = 20^o$, then $\angle AOC$ is equal to : **[1]**



a) 60^o b) 10^o

c) 40^o d) 20^o

4. In a trapezium ABCD, E and F be the midpoints of the diagonals AC and BD respectively. Then, EF = ? **[1]**



a) $\frac{1}{2}AB$

b) $\frac{1}{2}(AB+CD)$

c) $\frac{1}{2}(AB-CD)$

d) $\frac{1}{2}CD$

5. The value of $x^{p-q} x^{q-r} x^{r-p}$ is equal to **[1]**

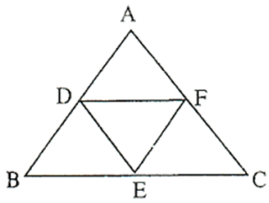
a) x^{pqr}

b) 0

c) x

d) 1

6. D, E and F are the mid points of sides AB, BC and CA of ΔABC. If perimetre of ΔABC is 16 cm, then perimetre of ΔDEF. **[1]**



a) 32 cm

b) 8 cm

c) 28 cm

d) 4 cm

7. x = 2, y = -1 is a solution of the linear equation **[1]**

a) 2x + y = 0

b) x + 2y = 0

c) x + 2y = 4

d) 2x + y = 5

8. If x - 3 is a factor of x^2 - ax -15, then a = **[1]**

a) 5

b) -2

c) -5

d) 3

9. The value of $15\sqrt{15} \div 3\sqrt{5}$ is **[1]**

a) $5\sqrt{3}$

b) $3\sqrt{5}$

c) 3

d) 5

10. ABCD is a parallelogram. If is produced to E such that ED bisects BC at O. Then which of the following is correct? **[1]**

a) OC = BE

b) OE = OC

c) AB = OE

d) AB = BE

11. An irrational number between $\frac{1}{7}\ and\ \frac{2}{7}$ is **[1]**

a) $\sqrt{\frac{1}{7} \times \frac{2}{7}}$

b) $\frac{1}{2}\left(\frac{1}{7} - \frac{2}{7}\right)$

c) $\left(\frac{1}{7} \times \frac{2}{7}\right)$

d) $\frac{1}{2}\left(\frac{1}{7} + \frac{2}{7}\right)$

12. How many linear equations in ‘x’ and ‘y’ can be satisfied by x = 1, y = 2? **[1]**

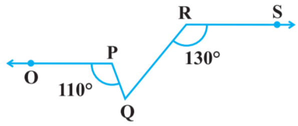
a) Infinitely many

b) Two

c) Only one

d) Three

13. In a figure, if OP||RS, ∠OPQ = 110° and ∠QRS = 130°, then ∠PQR is equal to **[1]**



a) 40°

b) 50°

c) 70°

d) 60°

14. After rationalising the denominator of $\frac{7}{3\sqrt{3}-2\sqrt{2}}$, we get the denominator as **[1]**

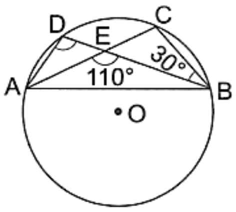
a) 5

b) 35

c) 19

d) 13

15. In the given figure, O is the centre of a circle and chords AC and BD intersect at E. If ∠AEB = 110° and ∠CBE = 30°, then ∠ADB = ? **[1]**



a) 80°

b) 60°

c) 90°

d) 70°

16. x co-ordinate is known as **[1]**

a) Origin

b) Points

c) Abscissa

d) Ordinate

17. If (-2, 5) is a solution of 2x + my = 11, then the value of 'm' is **[1]**

a) -2

b) 2

c) 3

d) -3

18. The value of $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$ is **[1]**

a) 3(a - b)(b - c)(c - a)

b) (a + b)(b + c)(c + a)

c) 3(a + b)(b + c)(c + a)(a - b)(b - c)(c - a)

d) 2(a - b)(b - c)(c - a)

19. **Assertion (A):** If the diagonals of a parallelogram ABCD are equal, then ∠ABC = 90^o **[1]**

Reason (R): If the diagonals of a parallelogram are equal, it becomes a rectangle.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $2+\sqrt{6}$ is an irrational number. **[1]**

Reason (R): Sum of a rational number and an irrational number is always an irrational number.

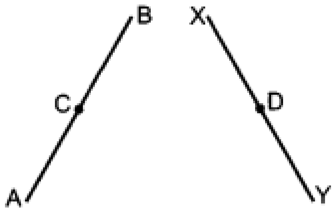
a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

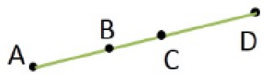
c) A is true but R is false. d) A is false but R is true.

Section B

21. In fig. AC = XD, C is the mid-point of AB and D is the mid-point of XY. Using a Euclid's axiom, show that AB = XY. **[2]**



22. In fig., if AC = BD, then prove that AB = CD **[2]**



23. Name the quadrants in which the following points lie : **[2]**

(i) p(4, 4)

(ii) Q(–4, 4)

(iii) R(–4, –4)

(iv) S(4, –4)

24. If x = 3 + $2\sqrt{2}$, find the value of $\left(x^2 + \frac{1}{x^2}\right)$. **[2]**

OR

Prove that: $\frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1} = 1$.

25. The radii of two cones are in the ratio 2 : 1 and their volumes are equal. What is the ratio of their heights? **[2]**

OR

A hollow spherical shell is made of a metal of density 4.5 g per cm^3. If its internal and external radii are 8 cm and 9 cm respectively, find the weight of the shell.

Section C

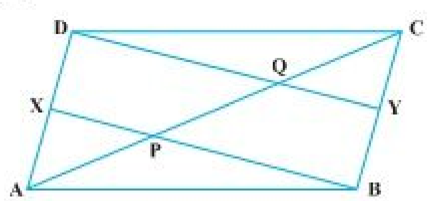
26. Locate $\sqrt{10}$ on the number line. **[3]**

27. A random survey of the number of children of various age groups playing in a park was found as follows : **[3]**

Age (in years)	**Number of children**
1-2	5
2-3	3
3-5	6
5-7	12
7-10	9
10-15	10
15-17	4

Draw a histogram to represent the data above.

28. In Fig. X and Y are respectively the mid-points of the opposite sides AD and BC of a parallelogram ABCD. Also, BX and DY intersect AC at P and Q, respectively. Show that AP = PQ = QC. **[3]**



29. Find the solution of the linear equation x + 2y = 8 which represents a point on **[3]**

  i. The x-axis
  ii. The y-axis

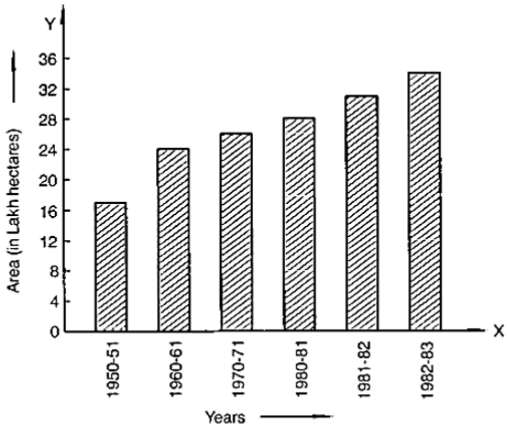
30. The marks scored by 750 students in an examination are given in the form of a frequency distribution table. **[3]**

Marks:	600-640	640-680	680-720	720-760	760-800	800-840	840-880
No. of Students:	16	45	156	284	172	59	18

Represent this data in the form of a histogram and construct a frequency polygon.

OR

Read the bar graph given in Figure and answer the following questions:

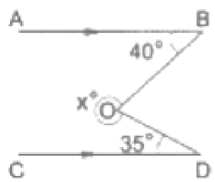


  i. What information is given by the bar graph?
  ii. In which years the areas under the sugarcane crop were the maximum and the minimum?
  iii. State whether true or false:
  The area under the sugarcane crop in the year 1982-83 is three times that of the year 1950-51.

31. If both (x- 2) and $\left(x-\frac{1}{2}\right)$ are factors of px^2+5x+r , Show that p = r. **[3]**

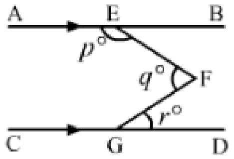
Section D

32. In the given figure, AB || CD, $\angle ABO = 40^\circ$, $\angle CDO = 35^\circ$. Find the value of the reflex ∠BOD and hence the value of x. **[5]**



OR

In the given figure, AB || CD. Prove that p + q - r = 180.



33. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? **[5]**

Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. (Use $\pi = 3.14$)

34. The length of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. Find the area of the triangle and the height corresponding to the longest side **[5]**

OR

Two sides of a triangular field are 85 m and 154 m in length and its perimeter is 324 m. Find the area of the field.

35. Using factor theorem, factorize the polynomial: $x^3 - 6x^2 + 3x + 10$ **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Peter, Kevin James, Reeta and Veena were students of Class 9th B at Govt Sr Sec School, Sector 5, Gurgaon. Once the teacher told **Peter to think a number x and to Kevin to think another number y** so that the difference of the numbers is 10 (x > y).

Now the teacher asked James to add double of Peter's number and that three times of Kevin's number, the total was found 120.

Reeta just entered in the class, she did not know any number.

The teacher said Reeta to form the 1st equation with two variables x and y.

Now Veena just entered the class so the teacher told her to form 2nd equation with two variables x and y.

Now teacher Told Reeta to find the values of x and y. Peter and kelvin were told to verify the numbers x and y.

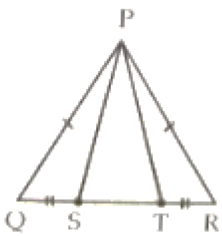
i. What are the equation formed by Reeta and Veena? (1)
ii. What was the equation formed by Veena? (1)
iii. Which number did Peter think? (2)

OR

Which number did Kelvin think? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

A children's park is in the shape of isosceles triangle said PQR with PQ = PR, S and T are points on QR such that QT = RS.



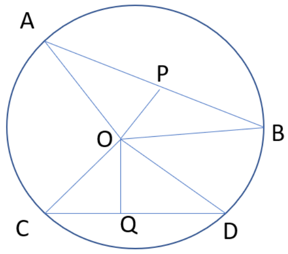
i. Which rule is applied to prove that congruency of $\triangle PQS$ and $\triangle PRT$. (1)
ii. Name the type of $\triangle PST$. (1)
iii. If PQ = 6 cm and QR = 7 cm, then find perimeter of $\triangle PQR$. (2)

OR

If $\angle QPR = 80^{o}$ find $\angle PQR$? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Rohan draws a circle of radius 10 cm with the help of a compass and scale. He also draws two chords, AB and CD in such a way that the perpendicular distance from the center to AB and CD are 6 cm and 8 cm respectively. Now, he has some doubts that are given below.



i. Show that the perpendicular drawn from the Centre of a circle to a chord bisects the chord. (1)
ii. What is the length of CD? (1)
iii. What is the length of AB? (2)

OR

How many circles can be drawn from given three noncollinear points? (2)

Solution

Section A

1.

(c) (3, 0)

Explanation: Since it lies on x-axis so ordinate will be zero because the value of the y-coordinate in the x-axis is equal to zero. Thus point will be (3, 0).

2.

(b) $10\sqrt{3}$ cm^2

Explanation: s = $\frac{5+7+8}{2}$= 10 cm

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{10(10-5)(10-7)(10-8)}$

= $\sqrt{10 \times 5 \times 3 \times 2}$

= $10\sqrt{3}$ sq. cm

3.

(c) 40^o

Explanation: Angle made by a chord at the centre is twice the angle made by it on any point of the circumference.

So, $\angle AOC = 2\angle ABC = 2 * 20^0 = 40^0$

4.

(c) $\frac{1}{2}(AB - CD)$

Explanation: Construction: Join CF and extent it to cut AB at point M

Firstly, in triangle MFB and triangle DFC

DF = FB (As F is the mid-point of DB)

$\angle$DFC = $\angle$MFB (Vertically opposite angle)

$\angle$DFC = $\angle$FBM (Alternate interior angle)

$\therefore$ By ASA congruence rule

$\triangle$MFB $\cong$ $\triangle$DFC

Now, in triangle CAM

E and F are the mid-points of AC and CM respectively

$\therefore$ EF = $\frac{1}{2}$ (AM)

EF = $\frac{1}{2}$ (AB – MB)

EF = $\frac{1}{2}$ (AB-CD)

5.

(d) 1

Explanation: $x^{p-q} x^{q-r} x^{r-p}$

$=x^{p-q+q-r+r-p}$

$= x^0$

=1

6.

(b) 8 cm

Explanation: Using relation

$perimeter.\Delta DEF = \frac{1}{2} perimeter.\Delta ABC$

$= \frac{1}{2} \times 16 = 8cm$

7.

(b) x + 2y = 0

Explanation: 2 + 2(-1) = 2 - 2 = 0

8.

(b) -2

Explanation: Put x - 3 = 0, then x = 3

Therefore, value of x^2-ax-15 at x=3 is zero

⇒ 3^2-3a-15=0

⇒ -6-3a=0

⇒ a=-2

9. **(a)** $5\sqrt{3}$

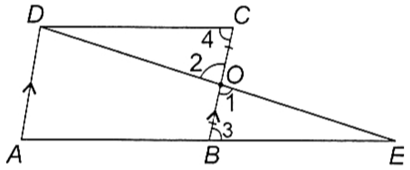
Explanation: $\frac{15\sqrt{15}}{3\sqrt{5}}$

$=\frac{(3\times5)\sqrt{3}\times\sqrt{5}}{3\sqrt{5}}$

$=5\sqrt{3}$

10.

(d) AB = BE

Explanation: 

In the figure, $\triangle$BCD is a parallelogram, where AB is produced to E such that OC = OB

In $\triangle$OBE and $\triangle$OCD,

$\angle$1 = $\angle$2 (Vertically opposite angles)

$\angle$3 = $\angle$4 (Alternate interior angles)

OB = OC (given)

$\therefore$ $\triangle$OBE $\cong$ $\triangle$OCD (By ASA congruency)

⇒ BE = CD (By CPCT)

Also, AB = CD (y ABCD is parallelogram)

$\therefore$ AB = BE

11. **(a)** $\sqrt{\frac{1}{7} \times \frac{2}{7}}$

Explanation: An irrational number between a and b is given by $\sqrt{ab}$.

So, an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$ is $\sqrt{\frac{1}{7} \times \frac{2}{7}}$.

12. **(a)** Infinitely many

Explanation: There are many linear equations in ‘x’ and ‘y’ can be satisfied by x = 1, y = 2

for example

x + y = 3 x - y =-1

2x + y =4

and so on there are infinte number of examples

13.

(d) 60°

Explanation: Produce OP to intersect RQ at point N.

Now, OP || RS and transversal RN intersects them at N and R respectively.

$\therefore \angle RNP = \angle SRN$ (Alternate interior angles)

⇒ $\angle$RNP = 130^o

$\therefore \angle PNQ = 180^\circ - 130^\circ = 50^\circ$ (Linear pair)

$\angle OPQ = \angle PNQ + \angle PQN$ (Exterior angle property)

⇒ 110^o = 50^o + $\angle PQN$

⇒ $\angle PQN$ = 110^o - 50^o = 60^o = $\angle PQR$

14.

(c) 19

Explanation: After rationalizing:

$\frac{7}{3\sqrt{3}-2\sqrt{2}} = \frac{7}{3\sqrt{3}-2\sqrt{2}} \times \frac{3\sqrt{3}+2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}}$
$= \frac{7(3\sqrt{3}+2\sqrt{2})}{(3\sqrt{3})^2-(2\sqrt{2})^2}$
$= \frac{7(3\sqrt{3}+2\sqrt{2})}{27-8}$
$= \frac{7(3\sqrt{3}+2\sqrt{2})}{19}$

15. **(a)** 80°

Explanation: We have:
$\angle$AEB + $\angle$CEB = 180° (Linear pair angles)
$\Rightarrow$ 110° + $\angle$CEB = 180°
$\Rightarrow$ $\angle$CEB = (180° - 110°) = 70°
$\Rightarrow$ $\angle$CEB = 70°
In $\triangle$CEB, we have:
$\angle$CEB + $\angle$EBC + $\angle$ECB = 180° (Angle sum property of a triangle)
$\Rightarrow$ 70° + 30° + $\angle$ECB = 180°
$\Rightarrow$ $\angle$ECB = (180° - 100°) = 80°
The angles in the same segment are equal.
Thus, $\angle$ADB = $\angle$ECB = 80°
$\Rightarrow$ $\angle$ADB = 80°

16.

(c) Abscissa
Explanation: Any point p in cartesian plane is written as p(x, y).
x coordinate of point p is called abscissa and Y co-ordinate of point p is called ordinate.

17.

(c) 3
Explanation: If (-2, 5) is a solution of 2x + my = 11
then it will satisfy the given equation
2 .(-2)+5 m=11
-4+5 m=11
5 m=11+4
5 m=15
$m = \frac{15}{5} = 3$
m=3

18.

(b) (a + b)(b + c)(c + a)
Explanation: $\frac{\left(a^2-b^2\right)^3+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$
$= \frac{3\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)}{3(a-b)(b-c)(c-a)}$ [Since $x^3 + y^3 + z^3 = 3xyz$, if x + y + z = 0]
$= \frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$
= (a + b)(b + c)(c + a)

19. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. In the above figure, we have
AB = AC + BC = AC + AC = 2AC (Since, C is the mid-point of AB) ..(1)
XY = XD + DY = XD + XD = 2XD (Since, D is the mid-point of XY) ..(2)
Also, AC = XD (Given) ..(3)

From (1),(2)and(3), we get

AB = XY, According to Euclid, things which are double of the same things are equal to one another.

22. AC = BD . . . . [Given] . . . (1)

AC = AB + BC . . . . [Point B lies between A and C] . . . . (2)

BD = BC + CD . . . . [Point C lies between B and D] . . . . (3)

Substituting (2) and (3) in (1), we get

AB + BC = BC + CD

⇒ AB = CD . . . . [Subtracting equals from equals]

23. (i) I

(ii) II

(iii) III

(iv) IV

24. Given, x=3+2 $\sqrt{2}$

$\therefore \quad \frac{1}{x}=\frac{1}{(3+2\sqrt{2})}$

= $\frac{1}{(3+2\sqrt{2})}\times\frac{(3-2\sqrt{2})}{(3-2\sqrt{2})}$

= $\frac{(3-2\sqrt{2})}{(3)^2-(2\sqrt{2})^2}$

= $\frac{(3-2\sqrt{2})}{(9-8)}$

= $3-2\sqrt{2}$

$\therefore \quad x+\frac{1}{x}=(3+2\sqrt{2})+(3-2\sqrt{2})$

x + $\frac{1}{x}$=6

$\Rightarrow \left(x+\frac{1}{x}\right)^2=6^2=36$

$\Rightarrow \quad x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}=36$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)+2=36 \Rightarrow \left(x^2+\frac{1}{x^2}\right)=36-2=34$

Hence, $\left(x^2+\frac{1}{x^2}\right)=34$

OR

LHS

$=\frac{1}{3+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+1}$

$=\frac{1}{3+\sqrt{7}}\times\frac{3-\sqrt{7}}{3-\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{5}}\times\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$

$=\frac{3-\sqrt{7}}{3^2-\sqrt{7}^2}+\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}^2-\sqrt{5}^2}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}^2-\sqrt{3}^2}+\frac{\sqrt{3}-1}{\sqrt{3}^2-1^2}$

$=\frac{3-\sqrt{7}}{9-7}+\frac{\sqrt{7}-\sqrt{5}}{7-5}+\frac{\sqrt{5}-\sqrt{3}}{5-3}+\frac{\sqrt{3}-1}{3-1}$

$=\frac{3-\sqrt{7}}{2}+\frac{\sqrt{7}-\sqrt{5}}{2}+\frac{\sqrt{5}-\sqrt{3}}{2}+\frac{\sqrt{3}-1}{2}$

$=\frac{3-\sqrt{7}+\sqrt{7}-\sqrt{5}+\sqrt{5}-\sqrt{3}+\sqrt{3}-1}{2}$

$=\frac{2}{2}$

= 1

= RHS

25. Radii of two cones are in the ratio of = 2 : 1

Let r_1, r_2 be the radii of two cones and h_1, h_2 be their respective heights .

Then $\frac{r_1}{r_2}=\frac{2}{1}$

Now, $\frac{Volume\ of first\ cone}{Volume\ of\ the\ second\ cone}$

$=\frac{\frac{1}{3}\pi r_1^2h_1}{\frac{1}{3}\pi r_2^2h_2}$

$=\frac{r_1^2h_1}{r_2^2h_2}=\left(\frac{r_1}{r_2}\right)^2\times\left(\frac{h_1}{h_2}\right)$

$=\left(\frac{2}{1}\right)^2\times\frac{h_1}{h_2}=\frac{4h_1}{h_2}$

∵ Their volumes are equal

$\therefore \frac{4h_1}{h_2}=1$

$\Rightarrow \frac{h_1}{h_2} = \frac{1}{4}$

$\therefore$ Their ratio is = 1 : 4

OR

Internal radius of the hollow spherical shell, r = 8 cm

External radius of the hollow spherical shell, R = 9 cm

Therefore, Volume of the shell = $\frac{4}{3}\pi\left(R^3 - r^3\right)$

= $\frac{4}{3}\pi\left(9^3 - 8^3\right)$

= $\frac{4}{3} \times \frac{22}{7} \times (729 - 512)$

= $\frac{4 \times 22 \times 217}{21}$

= $\frac{88 \times 31}{3}$

= $\frac{2728}{3}$ cm^3

Weight of the shell = volume of the shell $\times$ density per cubic cm

= $\frac{2728}{3} \times 4.5 \approx 4092$ g = 4.092 kg

Therefore Weight of the shell = 4.092 kg

Section C

26. We can write 10 as

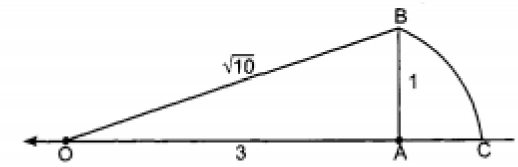
$10 = 9 + 1 = 3^2 + 1^2$

Draw OA = 3 units, on the number line

Draw BA = 1 unit, perpendicular to OA.

Join OB

Figure:



Now, by Pythagoras theorem,

$OB^2 = AB^2 + OA^2$

$OB^2 = 1^2 + 3^2 = 10$

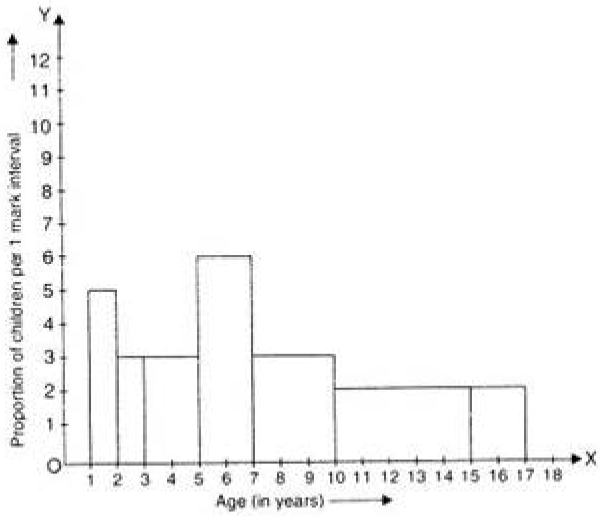
$\Rightarrow \quad OB = \sqrt{10}$

Taking O as centre and OB as a radius, draw an arc which intersects the number line at point C.

Clearly, OC corresponds to $\sqrt{10}$ on the number line.

27.

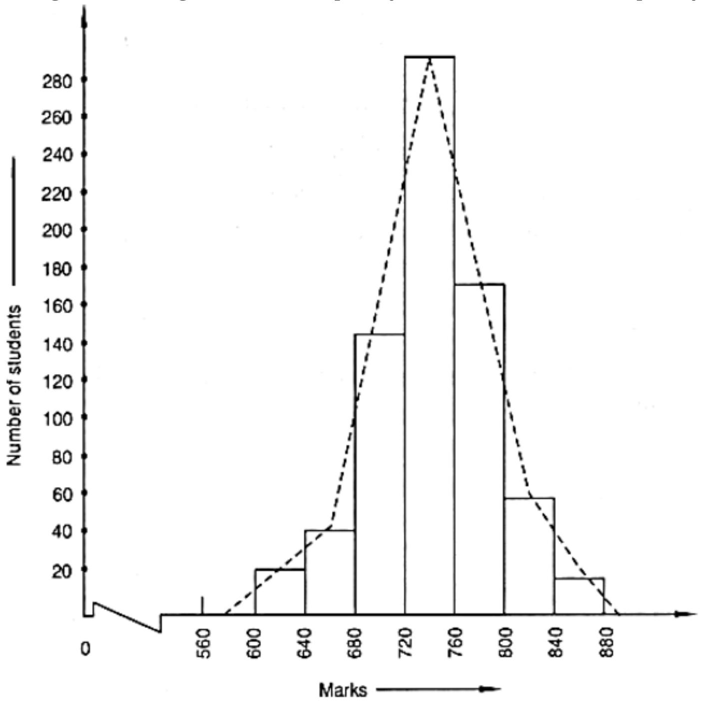
Age (in years)	Number of children(frequency)	Width of the class	Length of the rectangle
1-2	5	1	$\frac{5}{1} \times 1$ = 5
2-3	3	1	$\frac{3}{1} \times 1$ = 5
3-5	6	2	$\frac{6}{2} \times 1$ = 3
5-7	12	2	$\frac{12}{2} \times 1$ = 6
7-10	9	3	$\frac{9}{3} \times 1$ = 3
10-15	10	5	$\frac{10}{5} \times 1$ = 2
15-17	4	2	$\frac{4}{2} \times 1$= 2



28. AD = BC (Opposite sides of a parallelogram)
Therefore, $\mathrm{DX}=\mathrm{BY}\left(\frac{1}{2}\mathrm{AD}=\frac{1}{2}BC\right)$
Also, DX || BY (As AD || BC)
So, XBYD is a parallelogram (A pair of opposite sides equal and parallel)
i.e., PX || QD
Therefore, AP = PQ (From $\triangle$AQD where X is mid-point of AD) ...(1)
Similarly, from $\triangle$CPB, CQ = PQ ...(2)
Thus, AP = PQ = CQ [From (1) and (2)]

29. i. On x-axis y = 0
$\Rightarrow\ x+2\times 0=8\Rightarrow\ x=\ 8$
Therefore, the required point is (8, 0).
ii. On y-axis x = 0
$\Rightarrow\ 0+2y=8$
$\Rightarrow\ y=\frac{8}{2}\Rightarrow\ y=4$
Thus, the required point is (0, 4).

30. In Figure, a histogram and a frequency table of the above frequency distribution are drawn on the same scale.



To construct a frequency polygon without using the histogram of a given frequency distribution, we use the following algorithm.
STEP-I: Obtain the frequency distribution.
STEP-II: Compute the mid-points of class intervals i.e. class marks.
STEP-III: Represent class marks on X-axis on a suitable scale.
STEP-IV: Represent frequencies on Y-axis on a suitable scale.

STEP-V: Plot the points, where x denotes class mark and f corresponding frequency.

STEP-VI: Join the points plotted in step V by line segments.

STEP-VII: Take two class intervals of zero frequency, one at the beginning and the other at the end. Obtain their mid-points. These classes are known as imagined classes.

STEP-VIII: Complete the frequency polygon by joining the mid-points of first and last class intervals to the mid-points of the imagined classes adjacent to them.

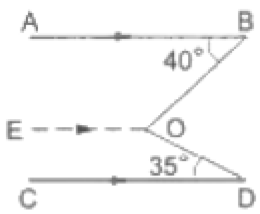
OR

i. It gives the information about the areas (in lakh hectors) under sugarcane crop during different years in India.
ii. The areas under the sugarcane crops were the maximum and minimum in 1982-83 and 1950-51 respectively.
iii. The area under sugarcane crop in the year 1982-83= 34 lakh hectares.
The area under sugarcane crop in the year 1950-51= 17 lakh hectares.
Clearly, the area under sugarcane crop in the year 1982-83 is not 3 times that of the year 1950-51
So, the given statement is false.

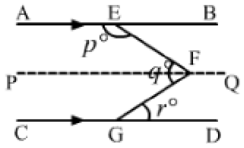
31. Suppose, $p(x) = px^2 + 5x + r$
As $(x-2)$ is a factor of $p(x)$
$\therefore p(2) = 0$
$\Rightarrow p(2)^2 + 5(2) + r = 0$
$\Rightarrow 4p + 10 + r = 0$ ...(1)
Again, $\left(x - \frac{1}{2}\right)$ is factor of p(x).
$\therefore p\left(\frac{1}{2}\right) = 0$
Now, $p\left(\frac{1}{2}\right) = p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r$
$= \frac{1}{4}p + \frac{5}{2} + r$
$\therefore p\left(\frac{1}{2}\right) = 0 \Rightarrow \frac{1}{4}p + \frac{5}{2} + r = 0$ ...(2)
From equation (1), we have $4p + r = -10$
From equation (2), we have $p + 10 + 4r = 0$
$\Rightarrow p + 4r = -10$
$\therefore 4p + r = p + 4r$ [$\because$ Each = - 10]
$\therefore 3p = 3r \Rightarrow p = r$
Hence, proved.

Section D

32. Through O, draw EO $\parallel$ AB $\parallel$ CD
Then, $\angle EOB + \angle EOD = x^\circ$,
Now, AB $\parallel$ EO and BO is the transversal
$\therefore \angle ABO + \angle BOE = 180^\circ$ [consecutive interior angles]
$\Rightarrow 40^\circ + \angle BOE = 180^\circ$
$\Rightarrow \angle BOE = (180^\circ - 40^\circ) = 140^\circ$
$\Rightarrow \angle$BOE =140°
Again CD $\parallel$ EO and OD is the transversal.
$\therefore \angle EOD + \angle ODC = 180^\circ$
$\Rightarrow \angle EOD + 35^\circ = 180^\circ$
$\Rightarrow \angle EOD = (180^\circ - 35^\circ) = 145^\circ$
$\Rightarrow \angle$EOD = 145°
$\therefore$ reflex $\angle BOD = x^\circ = (\angle BOE + \angle EOD)$
= (140° + 145°) = 285°
Hence, x° = 285°
$\Rightarrow \angle$BOD = x° = 285°



OR



Draw PFQ || AB || CD

Now, PFQ || AB and EF is the transversal.

Then,

$\angle AEF + \angle EFP = 180^\circ$ ...(i)

[Angles on the same side of a transversal line are supplementary]

Also, PFQ || CD.

$\angle PFG = \angle FGD = r^\circ$ [Alternate Angles]

and $\angle EFP = \angle EFG - \angle PFG = q^\circ - r^\circ$

putting the value of ∠EFP in equation (i)

we get,

p° + q °- r° = 180° [∠AEF = p°]

33. Height of the conical tent (h) = 8 m and Radius of the conical tent (r) = 6 m

Slant height of the tent $(l) = \sqrt{r^2 + h^2}$

$=\sqrt{(6)^2 + (8)^2}$

$=\sqrt{36 + 64}$

$=\sqrt{100}$

= 10 m

Area of tarpaulin = Curved surface area of tent = $\pi rl = 3.14 \times 6 \times 10 = 188.4\, m^2$

Width of tarpaulin = 3 m

Let Length of tarpaulin = L

∴ Area of tarpaulin = $Length \times Breadth = L \times 3$ = 3L

Now According to question, 3L = 188.4

⇒ L = 188.4/3 = 62.8 m

The extra length of the material required for stitching margins and cutting is 20 cm = 0.2 m.

So the total length of tarpaulin bought is (62.8 + 0.2) m = 63 m

34. Given, perimeter = 144 cm and ratio of the sides = 3 : 4 : 5

Sum of ratio terms = 3 + 4 + 5 = 12

∴ 1st side, a = 144 × $\frac{3}{12}$ = 36 cm

IInd side, b = 144 × $\frac{4}{12}$ = 48 cm

IIIrd side, c = 144 × $\frac{5}{12}$ = 60 cm

Now, semi-perimeter of the triangle,

s = $\frac{a+b+c}{2}$ = $\frac{36+48+60}{2}$ = $\frac{144}{2}$ = 72 cm

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ [by Heron's formula]

= $\sqrt{72 \times (72-36)(72-48)(72-60)}$

= $\sqrt{72 \times 36 \times 24 \times 12}$ = $\sqrt{(36)^2 \times (24)^2}$

= 36 × 24 = 864 cm^2

Hence, the area of the given triangle is 864 cm^2

Let height of a triangle be h cm.

Then, area of triangle = $\frac{1}{2}$ × Base × Height

⇒ 864 = $\frac{1}{2}$ × 60 × h [Since the longest side of a triangle is 60 cm, so we consider it as base of the triangle]

⇒ 864 = 30 h

⇒ h = 28.8 cm

Hence, the height corresponding to the longest side is 28.8 cm.

OR

Let:

a = 85 m and b = 154 m

Given that perimeter = 324 m

Perimeter= 2s = 324 m

$\Rightarrow$ s = $\frac{324}{2}$ m

or, a + b + c = 324

$\Rightarrow$ c = 324 - 85 - 154 = 85 m

By Herons's formula, we have:

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{162(162-85)(162-154)(162-85)}$

$= \sqrt{162 \times 77 \times 8 \times 77}$

$= \sqrt{1296 \times 77 \times 77}$

$= \sqrt{36 \times 77 \times 77 \times 36}$

$= 36 \times 77$

= 2772 m^2

35. Let, f(x) = $x^3 - 6x^2 + 3x + 10$

The constant term in f(x) is 10

The factors of 10 are $\pm$ 1, $\pm$ 2, $\pm$ 5, $\pm$ 10

Let, x + 1 = 0

$\Rightarrow$ x = -1

Substitute the value of x in f(x)

f(-1) = $(-1)^3$- $6(-1)^2$ + 3(−1) + 10

= -1 – 6 – 3 + 10

= 0

Similarly, (x – 2) and (x – 5) are other factors of f(x)

Since, f(x) is a polynomial having a degree 3, it cannot have more than three linear factors.

$\therefore$ f(x) = k(x + 1)(x – 2)(x – 5)

Substitute x = 0 on both sides

$\Rightarrow$ $x^3 - 6x^2 + 3x + 10$ = k(x + 1)(x – 2)(x – 5)

$\Rightarrow$ 0 – 0 + 0 + 10 = k(1)(-2)(-5)

$\Rightarrow$ 10 = k(10)

$\Rightarrow$ k = 1

Substitute k = 1 in f(x) = k(x + 1)(x – 2)(x – 5)

f(x) = (1)(x + 1)(x – 2)(x – 5)

so, $x^3 - 6x^2 + 3x + 10$ = (x + 1)(x – 2)(x – 5)

This is the required factorisation of f(x)

Section E

36. i. x - y = 10

2x + 3y = 120

ii. 2x + 3y = 120

iii. x - y = 10 ...(1)

2x + 3y = 120 ...(2)

Multiply equation (1) by 3 and to equation (2)

3x - 3y + 2x + 3y = 30 + 120

$\Rightarrow$ 5x = 150

$\Rightarrow$ x = 30

Hence the number thought by Prateek is 30.

OR

We know that x - y = 10 ...(i) and 2x + 3y = 120 ...(ii)

Put x = 30 in equation (i)

30 - y = 10

$\Rightarrow$ y = 40

Hence number thought by Kevin = 40.

37. i. In $\triangle$PQS and $\triangle$PRT

PQ = PR (Given)

QS = TR (Given)

$\angle$PQR = $\angle$ PRQ (corresponding angles of an isosceles $\triangle$)
By SAS commence
$\triangle$PQS $\cong$ $\triangle$PRT

ii. $\triangle PQS \cong \triangle PRT$
$\Rightarrow$ PS = PT (CPCT)
So in $\triangle$PST
PS = PT
It is an isosceles triangle.

iii. Perimeter = sum of all 3 sides
PQ = PR = 6 cm
QR = 7 cm
So, P = (6 + 6 + 7) cm
= 19 cm

OR

Let $\angle$Q = $\angle$R = x and $\angle$P = 80^o
In $\triangle$PQR, $\angle$P + $\angle$Q + $\angle$R = 180^o (Angle sum property of $\triangle$)
$80^o + x + x = 180^o$
$2x = 180^o - 80$
$2x = 100^o$
$x = \frac{100^\circ}{2}$
$= 50^o$

38. i. In ΔAOP and ΔBOP
$\angle$APO = $\angle$BPO (Given)
OP = OP (Common)
AO = OB (radius of circle)
ΔAOP $\cong$ ΔBOP
AP = BP (CPCT)

ii. In right ΔCOQ
$CO^2 = OQ^2 + CQ^2$
$\Rightarrow 10^2 = 8^2 + CQ^2$
$\Rightarrow CQ^2 = 100 - 64 = 36$
$\Rightarrow$ CQ = 6
CD = 2CQ
$\Rightarrow$ CD = 12 cm

iii. In right ΔAOB
$AO^2 = OP^2 + AP^2$
$\Rightarrow 10^2 = 6^2 + AP^2$
$\Rightarrow AP^2 = 100 - 36 = 64$
$\Rightarrow$ AP = 8
AB = 2AP
$\Rightarrow$ AB = 16 cm

OR

There is one and only one circle passing through three given non-collinear points.