Class 11 Maths Chapter 9 Exercise 9.4

Q1. Find the sum to n terms of the series
1 × 2 + 2 × 3 + 3 × 4 + 4 × 5 + ……………

Let S = 1 . 2 + 2 . 3 + 3 . 4 + 4 . 5 + …………….
Then, nth term,
$T_n = n(n + 1) = n^2 + n$
∴ $T_n = n^2 + n$
On taking summation from 1 to n on both sides we get

$$S_n = \sum_1^n T_n = \sum_1^n n^2 + \sum_1^n n$$
$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)}{6}[2n+1+3]$$
$$= \frac{n(n+1)}{6}[2n+4] = \frac{n(n+1)(n+2)}{3}$$

Q2.Find the sum to n terms of the series
1 × 2 × 3 + 2 × 3 × 4 + 3 × 4 × 5 + ………….

The given series is 1 × 2 × 3 + 2 × 3 × 4 + 3 × 4 × 5 + ………….
nth term $a_n = n (n + 1) (n + 2)$
$= (n^2 + n) (n + 2) = n^2 + 3n^2 + 2n$

$$S_n = \sum_{k=1}^n a_k = \sum_{k=1}^n k^3 + 3\sum_{k=1}^n k^2 + 2\sum_{k=1}^n k$$
$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{3n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$$
$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{2} + n(n+1)$$
$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + 2n + 1 + 2\right] = \frac{n(n+1)}{2}\left[\frac{n^2 + n + 4n + 6}{2}\right]$$
$$= \frac{n(n+1)}{4}(n^2 + 5n + 6) = \frac{n(n+1)}{4}(n^2 + 2n + 3n + 6)$$
$$= \frac{n(n+1)[n(n+2) + 3(n+2)]}{4} = \frac{n(n+1)(n+2)(n+3)}{4}$$

Q3. Find the sum of n terms of the series $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots\ldots\ldots\ldots\ldots$

The given series is $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots\ldots\ldots\ldots\ldots$
nth term $a_n = (2n + 1)\, n^2$
$= 2n^3 + n^2$
$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} (2k^3 + k^2) = 2\sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} k^2$
$= 2\left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$

$$= \frac{n^2 (n+1)^2}{2} + \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)}{2} + \left[n(n+1) + \frac{2n+1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2 + 3n + 2n + 1}{3}\right] = \frac{n(n+1)}{2}\left[\frac{3n^2 + 5n + 1}{3}\right]$$

$$= \frac{n(n+1)(3n^2 + 5n + 1)}{6}.$$

Q4. Find the sum to n terms of the series $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots\ldots$

Let the given series be
$S = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots\ldots$
Then, n^{th} term $T_n = \frac{1}{n(n+1)}$
Now, we will split the denominator of the n^{th} term into two parts or we will write T_n as the difference of two terms.

$$\therefore \quad T_n = \frac{1}{n(n+1)} = \frac{(n+1) - n}{n(n+1)}$$

$$= \frac{n+1}{n(n+1)} - \frac{n}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

On putting $n = 1, 2, 3, 4, \ldots$ successively, we get

$$T_1 = \frac{1}{1} - \frac{1}{2}$$

$$T_2 = \frac{1}{2} - \frac{1}{3}$$

$$T_3 = \frac{1}{3} - \frac{1}{4}$$

....

....

$$T_n = \frac{1}{n} = \frac{1}{n+1}$$

On adding all these terms, we get

$$S = T_1 + T_2 + T_3 + \ldots + T_n$$
$$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \ldots + \left(\frac{1}{n} + \frac{1}{n+1}\right)$$
$$= 1 - \frac{1}{n+1} = \frac{1}{1} - \frac{1}{n+1} = \frac{n+1-1}{n+1}$$

$\Rightarrow$ $$S = \frac{n}{n+1}$$

i.e., if $$T_n = \frac{(n+1)}{n^2}$$

Then, $$\Sigma T_n \neq \frac{\Sigma(n+1)}{\Sigma n^2}$$

Q5. Find the sum to n terms of the series $5^2 + 6^2 + 7^2 + \ldots + 20^2$.

The given series is $5^2 + 6^2 + 7^2 + \ldots + 20^2$

n^{th} term, $a_n = (n + 4)^2 = n^2 + 8n + 16$

$$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \left(k^2 + 8k + 16\right)$$
$$= \sum_{k=1}^{n} k^2 + 8\sum_{k=1}^{n} k + \sum_{k=1}^{n} 16$$
$$= \frac{n(n+1)(2n+1)}{6} + \frac{8n(n+1)}{2} + 16n$$

16^{th} term is $(16 + 4)^2 = 20^2 2$

$$\therefore S_{16} = \frac{16(16+1)(2\times16+1)}{6} + \frac{8\times16\times(16+1)}{2} + 16\times16$$
$$= \frac{(16)(17)(33)}{6} + \frac{(8)\times16\times(16+1)}{2} + 16\times16$$
$$= \frac{(16)(17)(33)}{6} + \frac{(8)(16)(17)}{2} + 256$$
$$= 1496 + 1088 + 256$$
$$= 2840$$
$$\therefore 5^2 + 6^2 + 7^2 + \ldots\ldots + 20^2 = 2840$$

Q6. Find the sum to n terms of the series 3 × 8 + 6 × 11 + 9 × 14 + …………..

The given series is 3 × 8 + 6 × 11 + 9 × 14 + …………
a_n = (nth term of 3, 6, 9 ………..) × (nth term of 8, 11, 14, …………)
= (3n) (3n + 5)
= $9n^2 + 15n$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} (9k^2 + 15k) = 9\sum_{k=1}^{n} k^2 + 15\sum_{k=1}^{n} k$$
$$= 9 \times \frac{n(n+1)(2n+1)}{6} + 15 \times \frac{n(n+1)}{2}$$
$$= \frac{3n(n+1)(2n+1)}{2} + \frac{15n(n+1)}{2} = \frac{3n(n+1)}{2}(2n+1+5)$$
$$= \frac{3n(n+1)}{2}(2n+6) = 3n(n+1)(n+3).$$

Q7. Find the sum to n terms of the series $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) +$ …………

The given series is $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^3) + \ldots$

$a_n = (1^2 + 2^2 + 3^3 + \ldots\ldots + n^2)$

$$= \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{n(2n^2+3n+1)}{6} = \frac{2^3+3n^2+n}{6}$$
$$= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

Q8. Find the sum to re terms of the series whose nth term is given by n (n + 1) (n + 4).

a_n = n (n + 1) (n + 4)
= $n(n^2 + 5n + 4)$
= $n^3 + 5n^2 + 4n$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^3 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k$$

$$= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{5(2n+1)}{3} + 4\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2 + 3n + 20n + 10 + 24}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2 + 23n + 34}{6}\right]$$

$$= \frac{n(n+1)(3n^2 + 23n + 34)}{12}$$

Q.9 Find the sum to n^{th} terms of the series whose nth term is given by the $n^2 + 2n$.

$a_n = n^2 + 2^n$

$$\therefore S_n = \sum_{k=1}^{n} k^2 + 2^k = \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 2^k \qquad (1)$$

Consider $\sum_{k=1}^{n} 2^k = 2^1 + 2^2 + 2^3 + \ldots$

The above series 2, 2^2, 2^3, ... is a G.P. with both the first term and common ratio equal to 2.

$$\therefore \sum_{k=1}^{n} 2^k = \frac{(2)\left[(2)^n - 1\right]}{2-1} = 2\left(2^n - 1\right) \qquad (2)$$

Therefore, from (1) and (2), we obtain

$$S_n = \sum_{k=1}^{n} k^2 + 2\left(2^n - 1\right) = \frac{n(n+1)(2n+1)}{6} + 2\left(2^n - 1\right)$$

Q10. Find the sum to re terms of the series whose n^{th} term is given by $(2n - 1)^2$.

Given, nth term $T_n = (2n - 1)^2$

$\Rightarrow T_n = 4n^2 + 1 - 4n$

Now, S = Σ T_n

= Σ ($4n^2$ + 1 – 4n)

= 4 Σn^2 + Σ 1 – 4 Σn

$$= \frac{4n(n+1)(2n+1)}{6} + n - \frac{4n(n+1)}{2}$$

$$\left[\because \Sigma 1 = n, \Sigma n = \frac{n(n+1)}{2} \text{ and } \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$$= n\left[\frac{2(n+1)(2n+1)}{3} + \frac{1}{1} - \frac{2(n+1)}{1}\right]$$

$$= n\left[\frac{2(2n^2+n+2n+1)+3-6(n+1)}{3}\right]$$

$$= \frac{n[4n^2+6n+2+3-6n-6]}{3}$$

$$= \frac{n(4n^2-1)}{3} = \frac{n}{3}(2n+1)(2n-1) \quad [\because (a^2-b^2) = (a-b)(a+b)]$$

$$\therefore S_n = \sum_{k=1}^{n} a_k$$

$$= \sum_{k=1}^{n}\left(\frac{1}{3}k^3 + \frac{1}{2}k^2 + \frac{1}{6}k\right)$$

$$= \frac{1}{3}\sum_{k=1}^{n}k^3 + \frac{1}{2}\sum_{k=1}^{n}k^2 + \frac{1}{6}\sum_{k=1}^{n}k$$

$$= \frac{1}{3}\frac{n^2\left(n+1\right)^2}{\left(2\right)^2} + \frac{1}{2}\times\frac{n\left(n+1\right)\left(2n+1\right)}{6} + \frac{1}{6}\times\frac{n\left(n+1\right)}{2}$$

$$= \frac{n\left(n+1\right)}{6}\left[\frac{n\left(n+1\right)}{2} + \frac{\left(2n+1\right)}{2} + \frac{1}{2}\right]$$

$$= \frac{n\left(n+1\right)}{6}\left[\frac{n^2+n+2n+1+1}{2}\right]$$

$$= \frac{n\left(n+1\right)}{6}\left[\frac{n^2+n+2n+2}{2}\right]$$

$$= \frac{n\left(n+1\right)}{6}\left[\frac{n\left(n+1\right)+2\left(n+1\right)}{2}\right]$$

$$= \frac{n\left(n+1\right)}{6}\left[\frac{\left(n+1\right)\left(n+2\right)}{2}\right]$$

$$= \frac{n\left(n+1\right)^2\left(n+2\right)}{12}$$